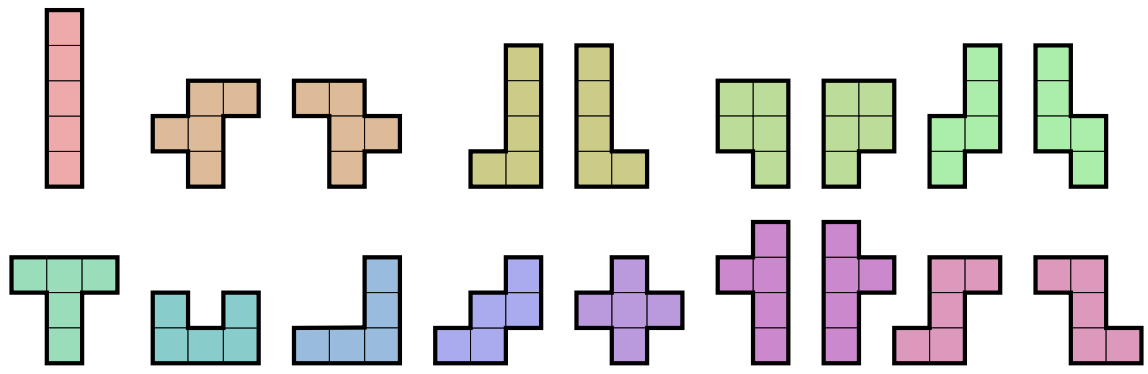
The twisted nature of polyominoes

Alex Vlasiuk



August 5, 2019

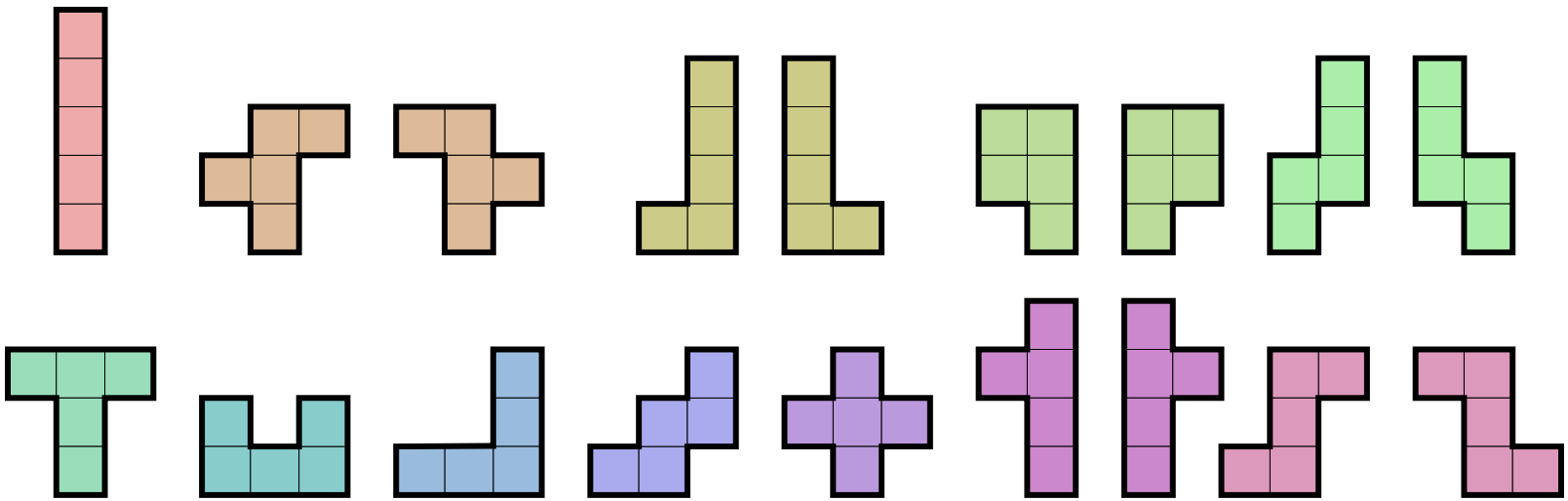
Poly-o-mino

Poly-o-mino

polyomino [pɒlɪˈɒmɪnəʊ] noun. Pl. **-oes**. *M20*.
ORIGIN: from poly- + domino.
A planar shape formed by joining a number of identical squares by their edges.

from *The Shorter Oxford English Dictionary*

Poly-o-mino



Dominoes

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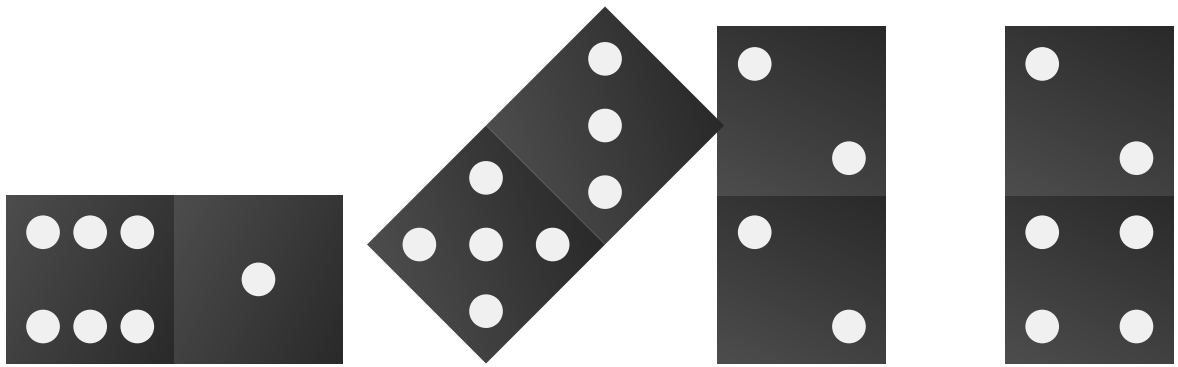
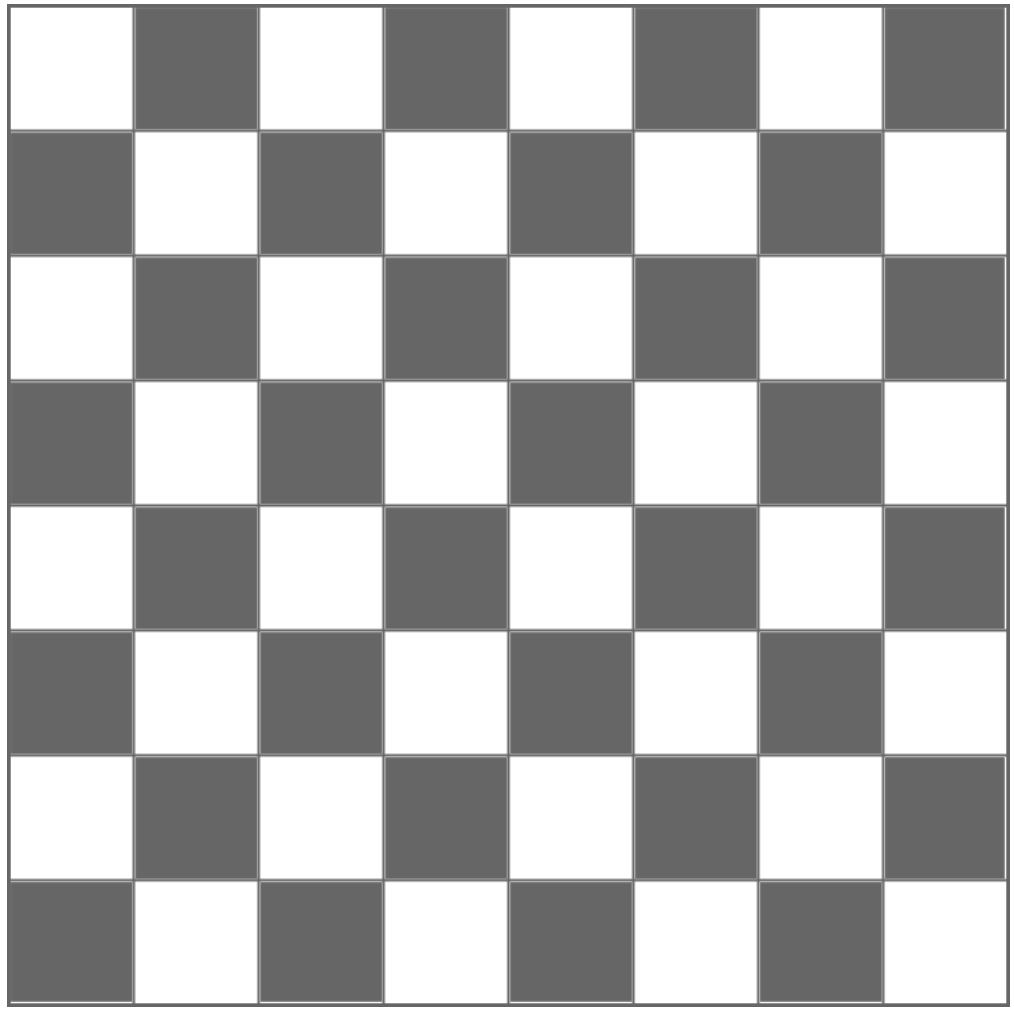


Dominoes & Chessboard I



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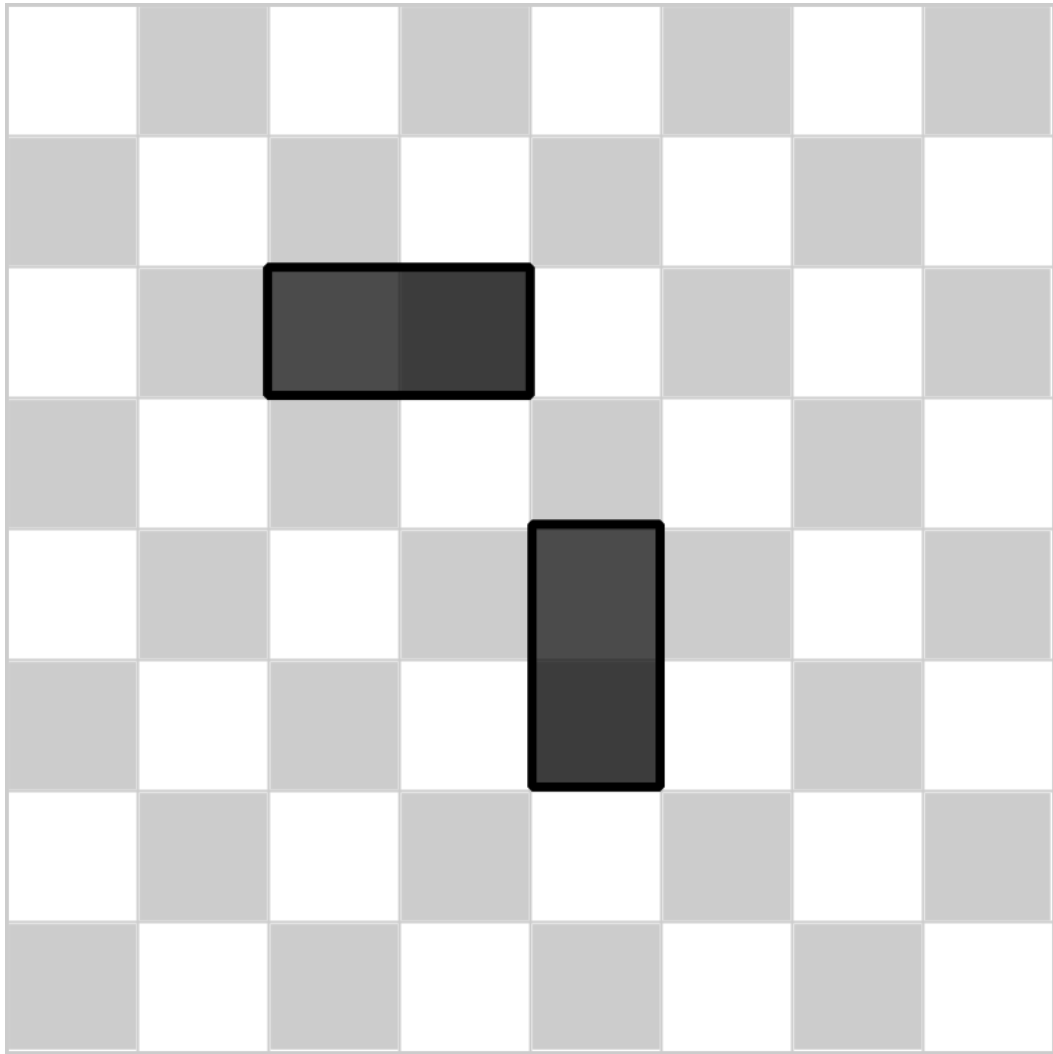

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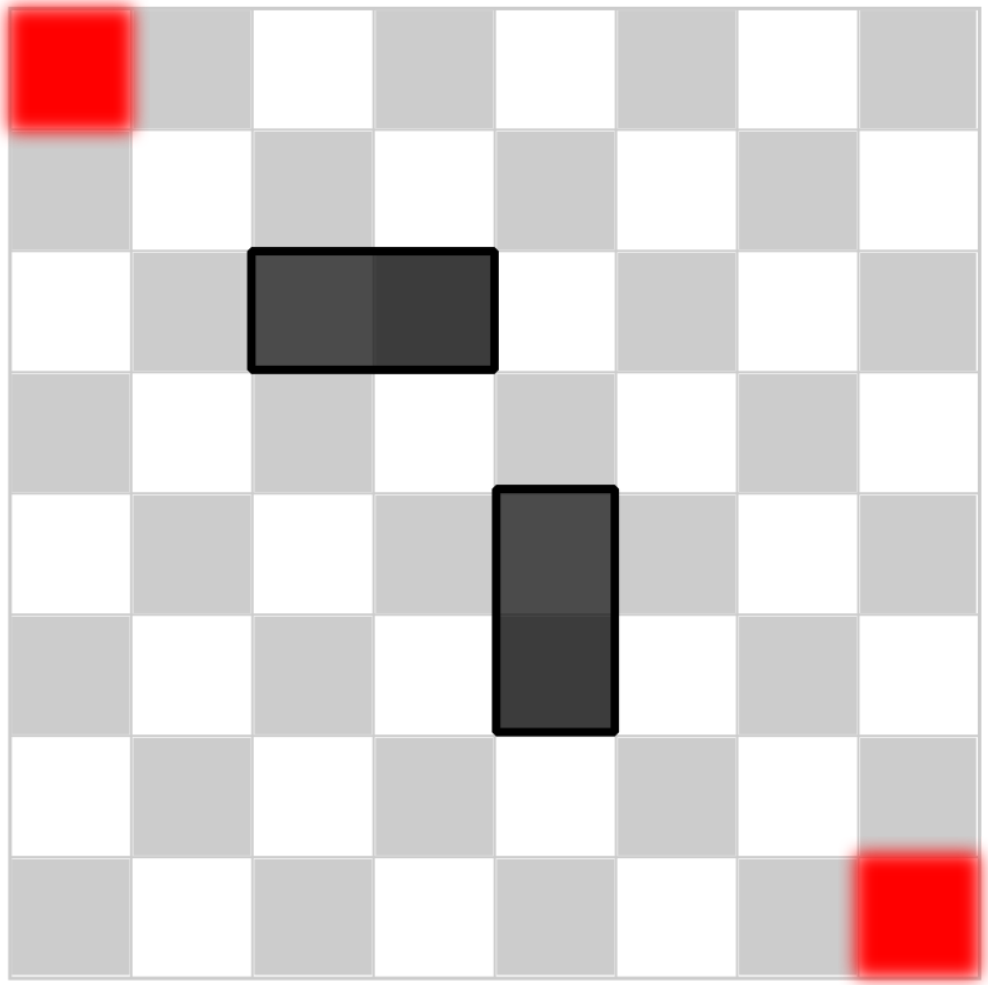

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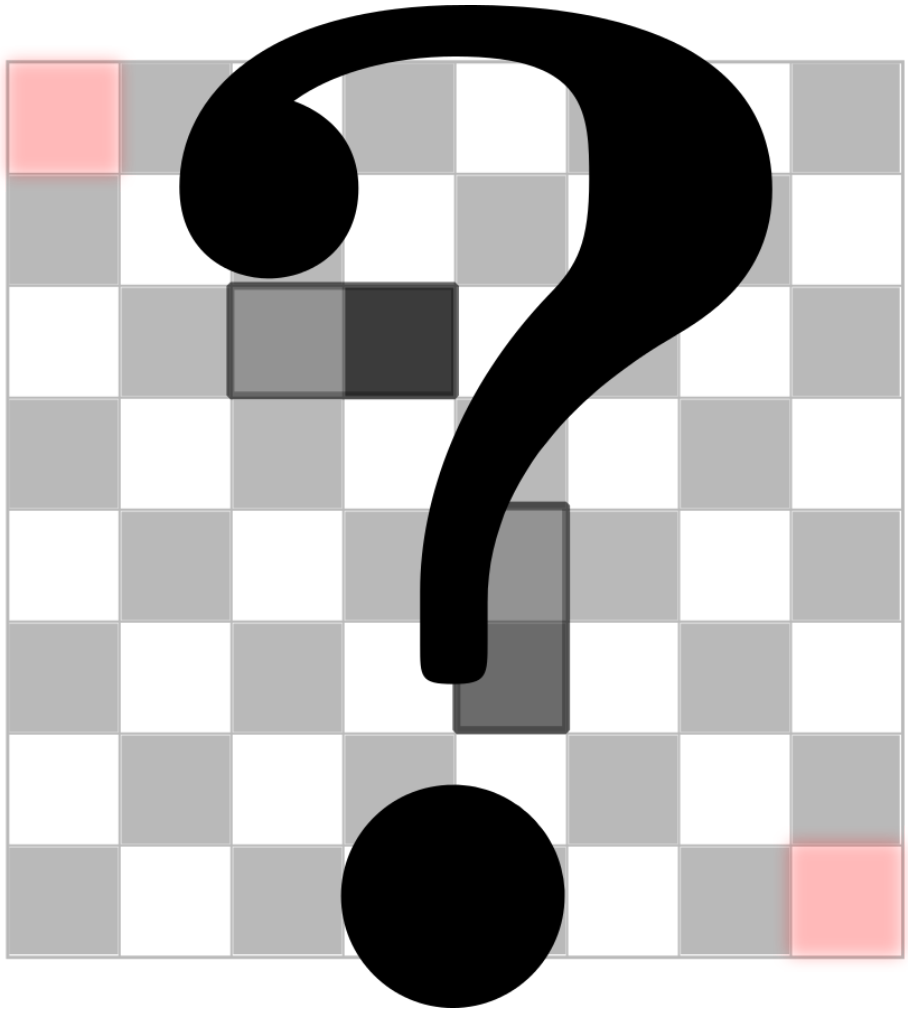
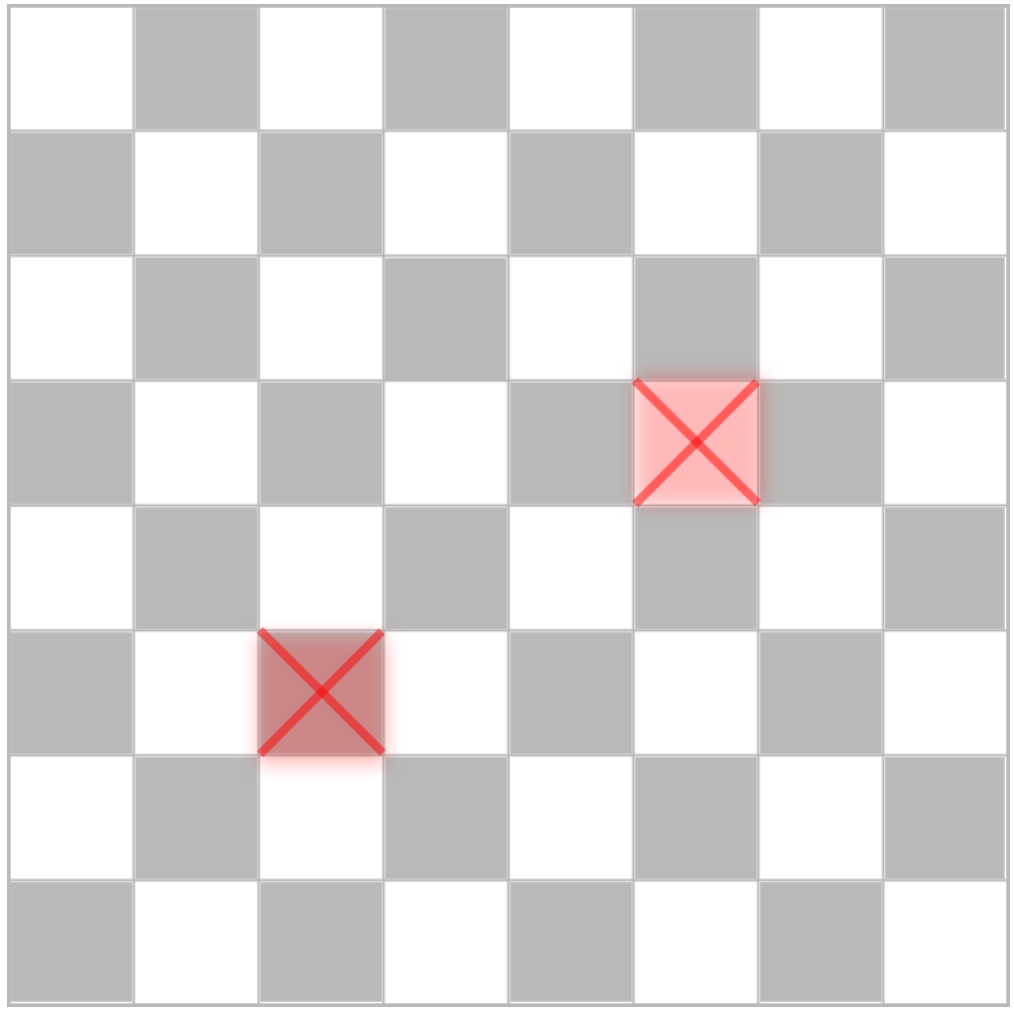
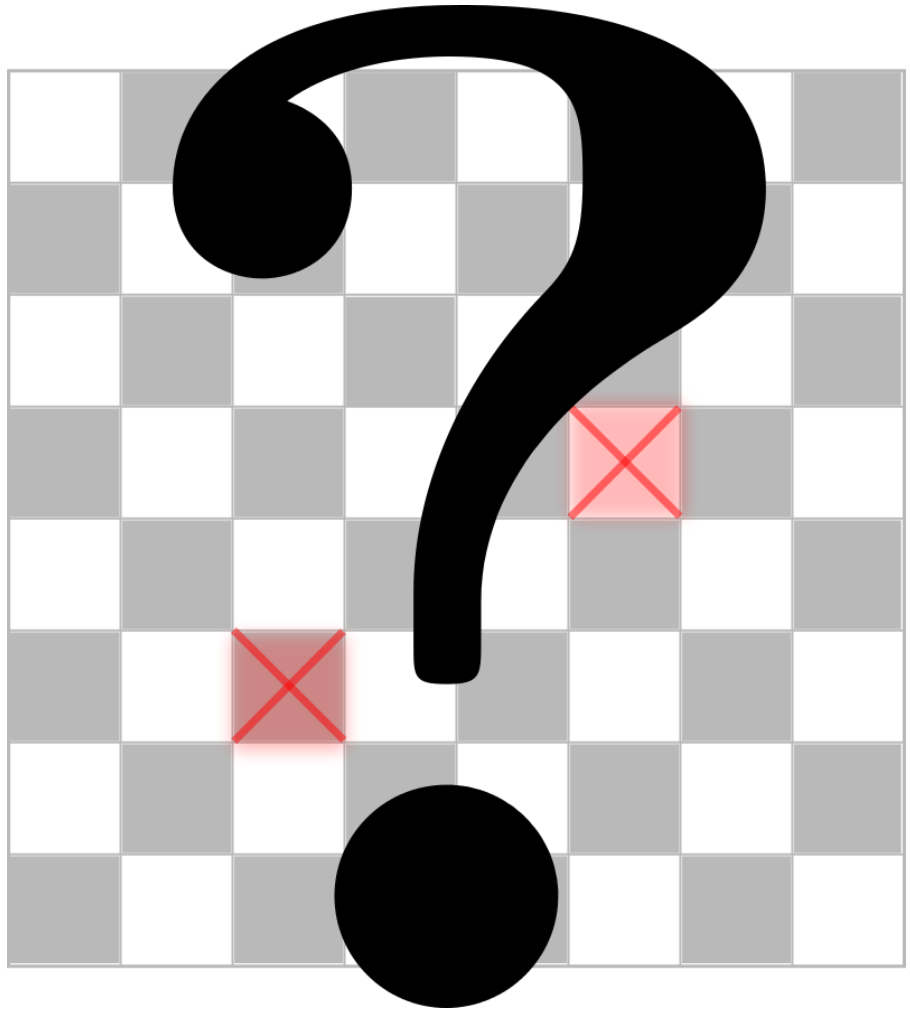


Dominoes & Chessboard II

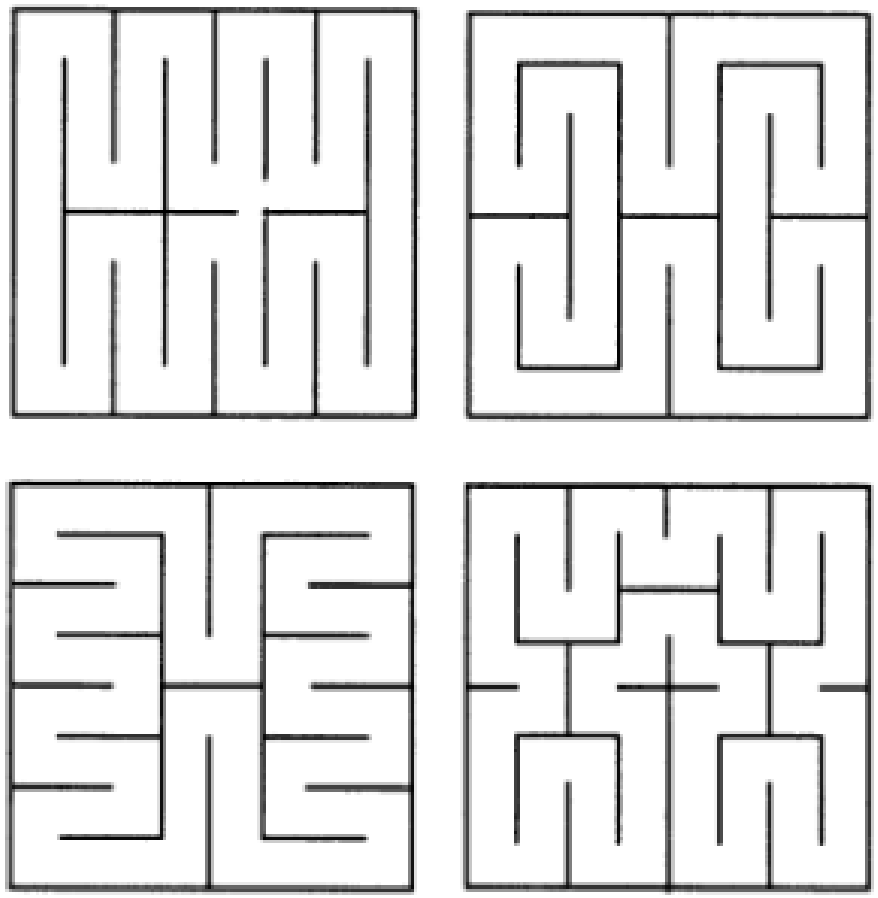


Dominoes & Chessboard II

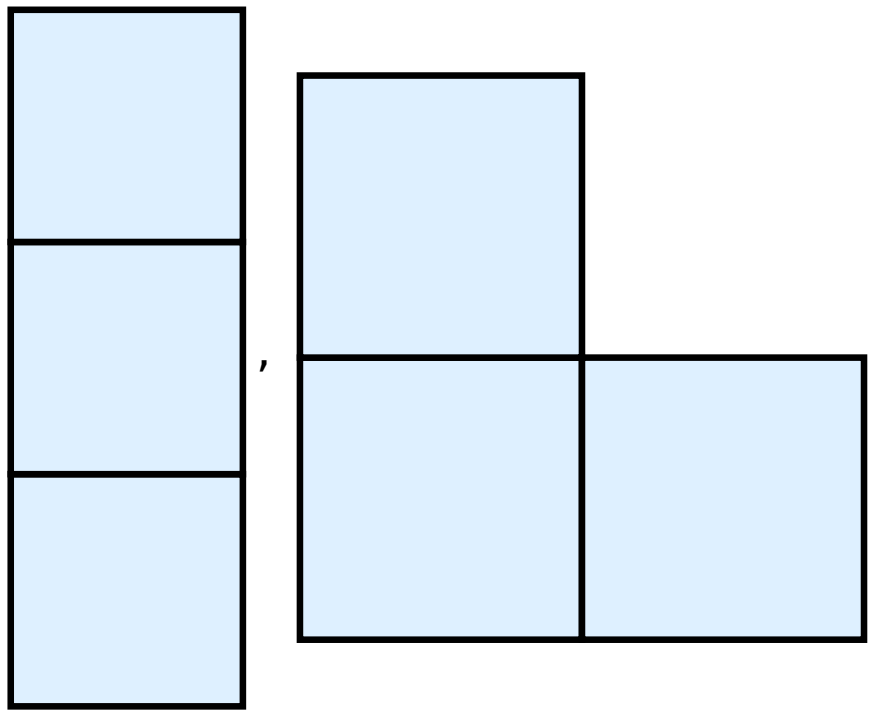


Dominoes & Chessboard II

Ralph Gomory's barriers



Trominoes



Golomb's Tromino Theorem

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Theorem (S.Golomb)

For any integer $N \geq 0$, if we remove a single square from a chess board of size $2^N \times 2^N$, the remaining board can entirely be tiled by basic trominos.

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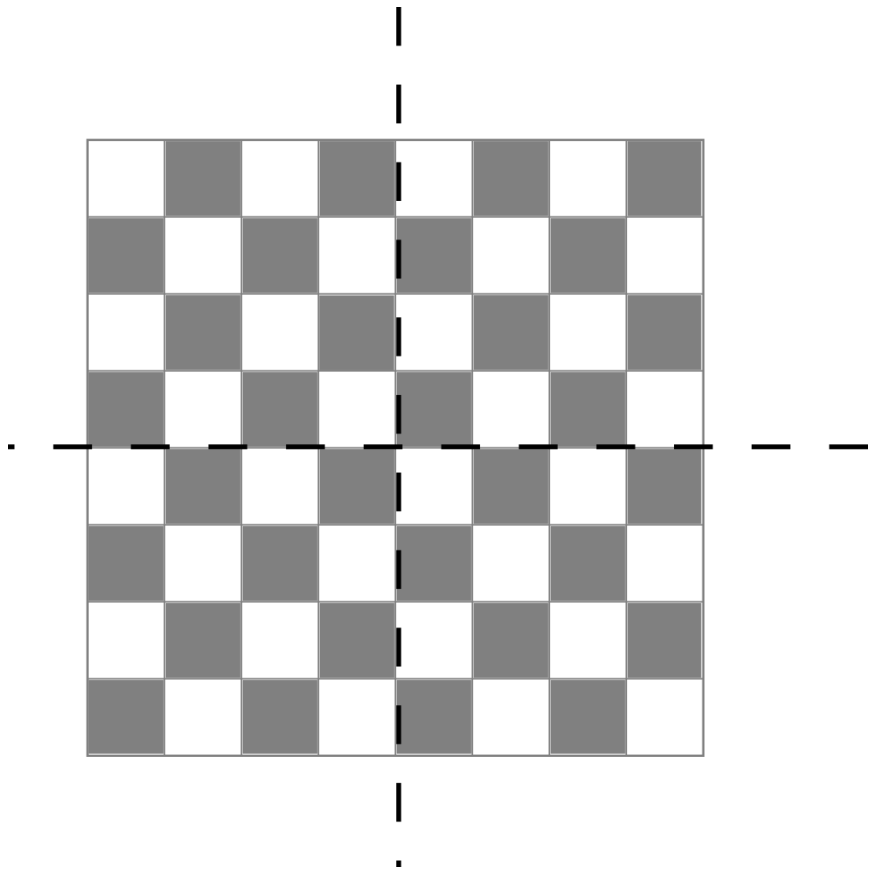

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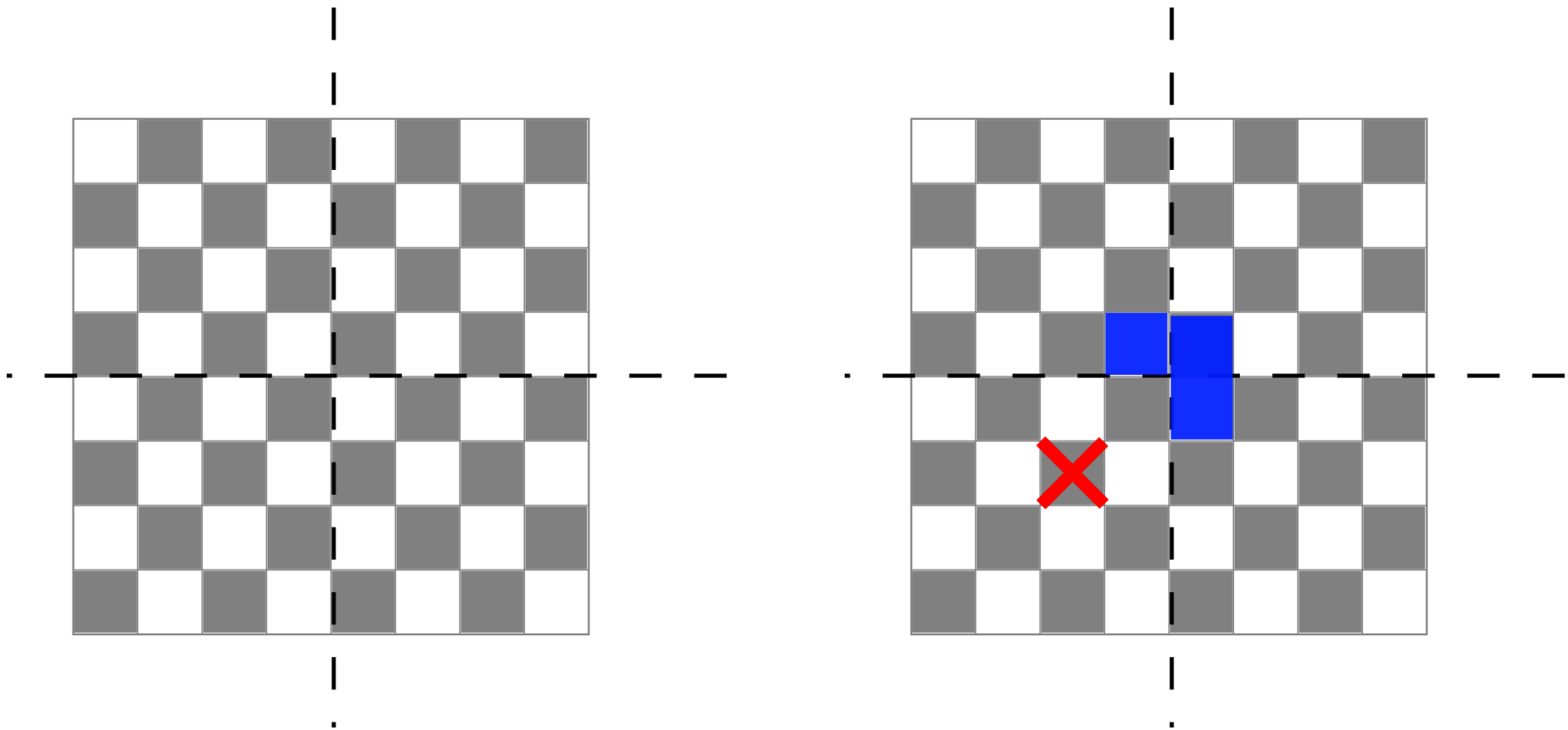

Tetrominoes

Tetrominous

The Origin of Evil


super
F1 MUSIC
SPEED
LEVEL
START / PAUSE
ON / OFF
MUTE
LEFT
LEVEL
RIGHT
SPEED
ROTATE
DIRECTION
DOWN
MODE
TALKING
BRICK
GAME
132 IN 1
E-132T


super
MUSIC
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START / PAUSE
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- Alexey Pajitnov, June 1984

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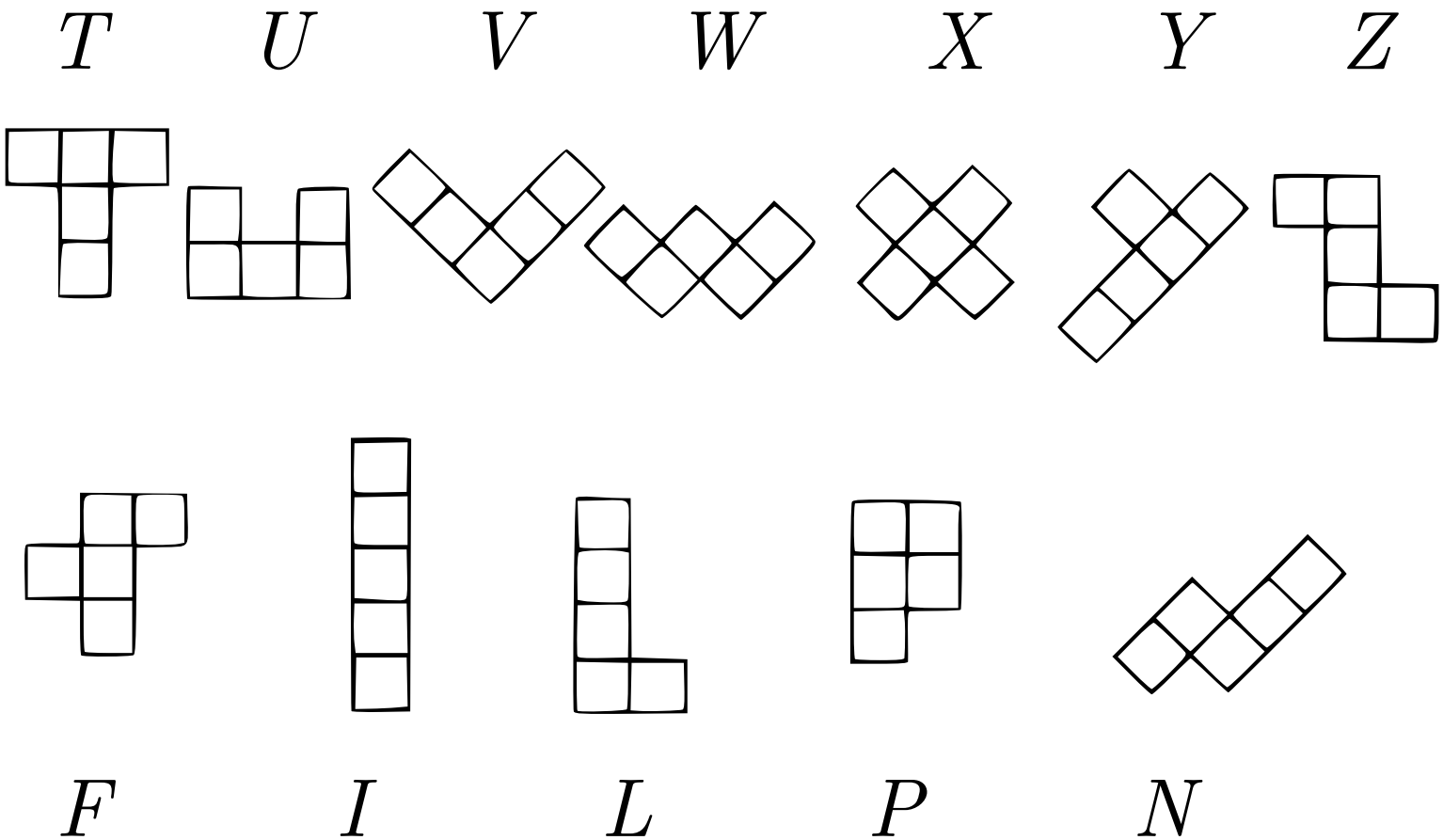
- Pajitnov said, Nintendo version is the best!

Pentominoes

Pentominoes



Grouping Pentominoes

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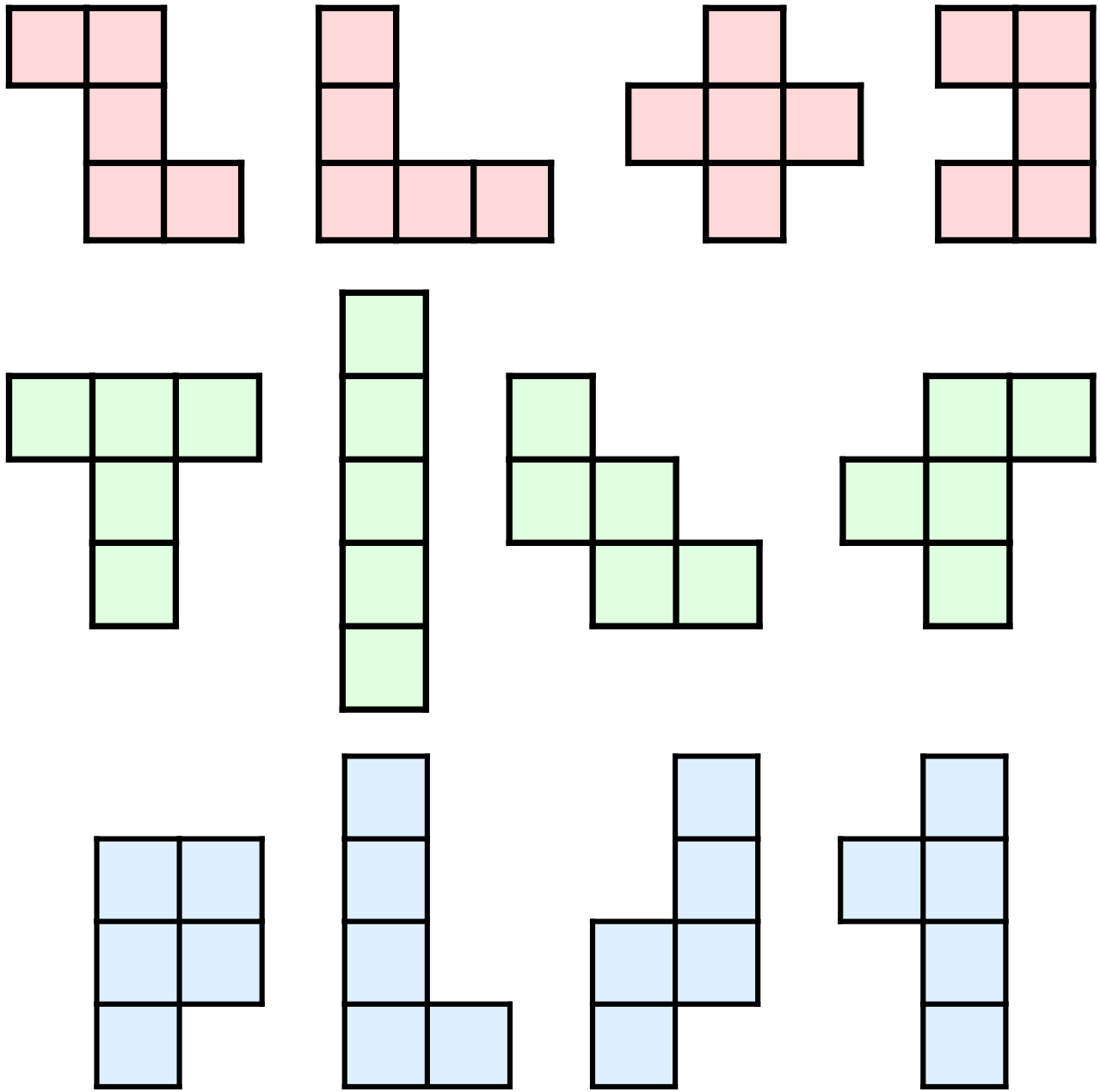

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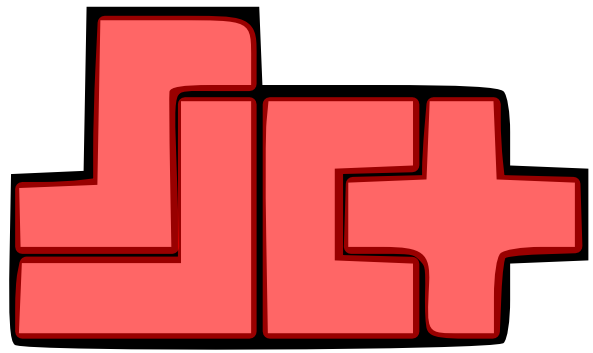
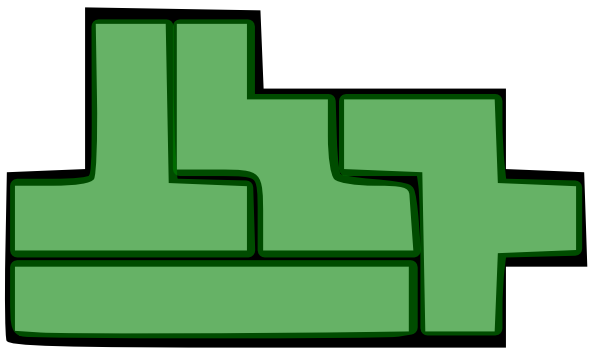
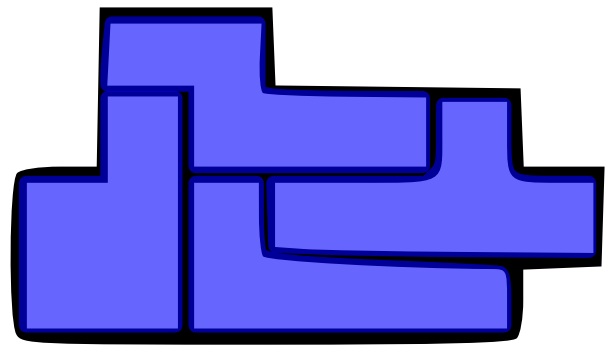


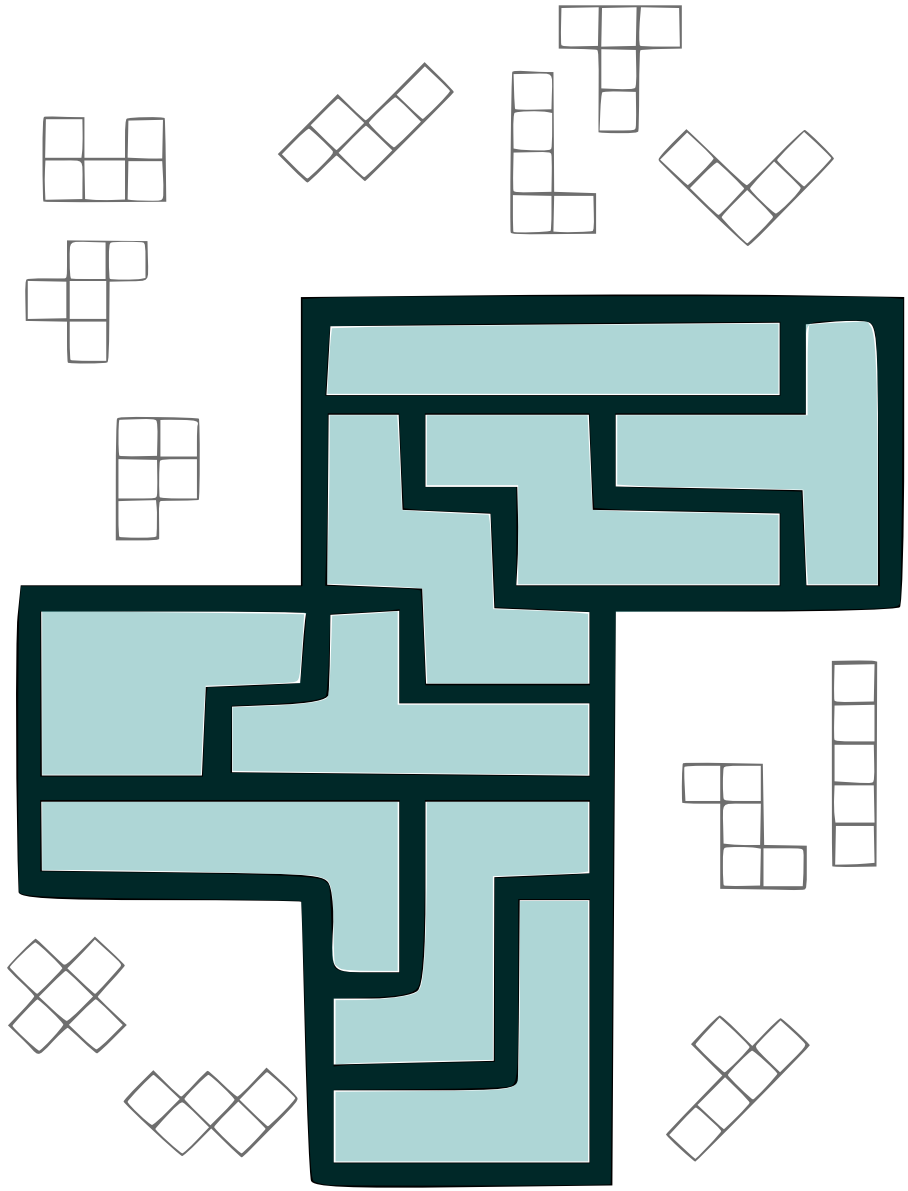




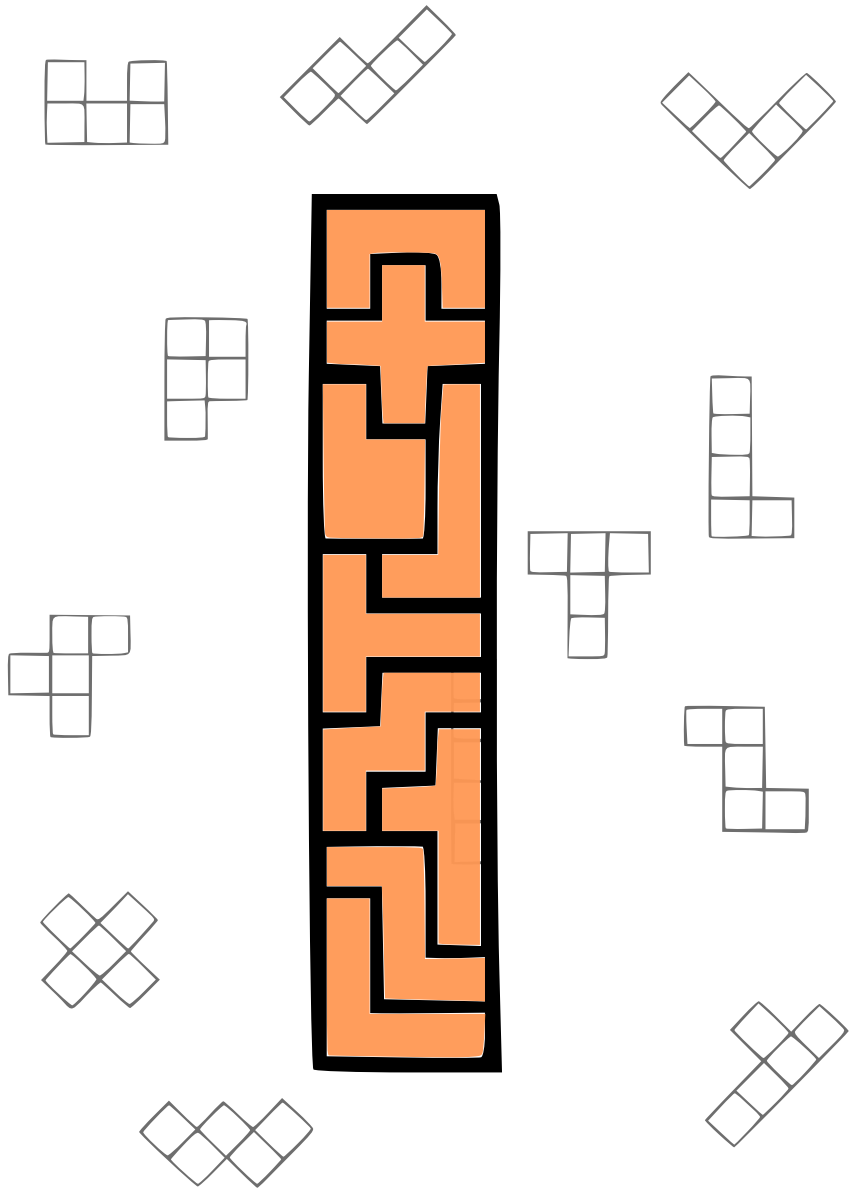
Golomb's Problem

[...] *the readers are asked to use nine complete sets of pentominoes to construct a set of models of the pentominoes three times as long and as wide.* **[...]** *the nine pieces used to construct each model must not contain duplicates, and must not contain the piece corresponding to the model.*

Golomb's Problem: F



Golomb's Problem: I



Counting One-sided Polyominoes

n	One-sided polyominoes
1	1
2	1
3	2
4	7
5	18
6	60
7	196
8	704
9	2500
10	9189
11	33896
12	126759
13	476270
14	1802312
15	6849777

n	One-sided polyominoes
16	26152418
17	100203194
18	385221143
19	1485200848
20	5741256764
21	22245940545
22	86383382827
23	336093325058
24	1309998125640
25	5114451441106
26	19998172734786
27	78306011677182
28	307022182222506
29	1205243866707468
30	4736694001644862

`OEIS: A000988`

Counting Free Polyominoes

n	Number of free polyominoes
1	1
2	1
3	2
4	5
5	12
6	35
7	108
8	369
9	1285
10	4655
11	17073
12	63600
13	238591
14	901971

n	Number of free polyominoes
15	3426576
16	13079255
17	50107909
18	192622052
19	742624232
20	2870671950
21	11123060678
22	43191857688
23	168047007728
24	654999700403
25	2557227044764
26	9999088822075
27	39153010938487
28	153511100594603

`OEIS: A000105`

THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES®

founded in 1964 by N. J. A. Sloane

Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

A000105 Number of free polyominoes (or square animals) with n cells. 92

(Formerly M1425 N0561)

1, 1, 1, 2, 5, 12, 35, 108, 369, 1285, 4655, 17073, 63600, 238591, 901971, 3426576, 13079255, 50107909, 192622052, 742624232, 2870671950, 11123060678, 43191857688, 168047007728, 654999700403, 2557227044764, 9999088822075, 39153010938487, 153511100594603

(list; graph; refs; listen; history; text; internal format)

OFFSET 0,4

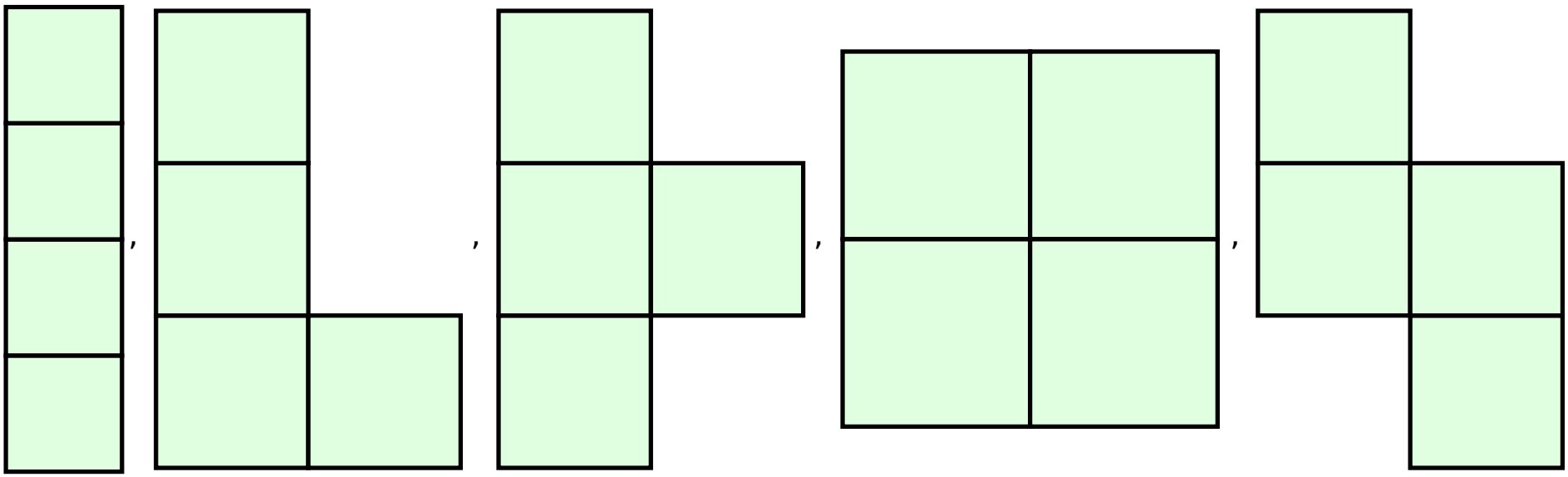
COMMENTS a(n) + A030228(n) = A000988(n) because the number of free polyominoes plus the number of polyominoes lacking bilateral symmetry equals the number of one-sided polyominoes. - Graeme McRae, Jan 05 2006

The possible symmetry groups of a (nonempty) polyomino are the 10 subgroups of the dihedral group D_8 of order 8: D_8, 1, Z_2 (five times), Z_4, (Z_2)^2 (twice). - Benoit Jubin, Dec 30 2008

Names for first few polyominoes: monomino, domino, tromino, tetromino, pentomino, hexomino, heptomino, octomino, enneomino, decomino, hendecomino, dodecomino, ...

Free Tetrominoes

Free Tetrominoes



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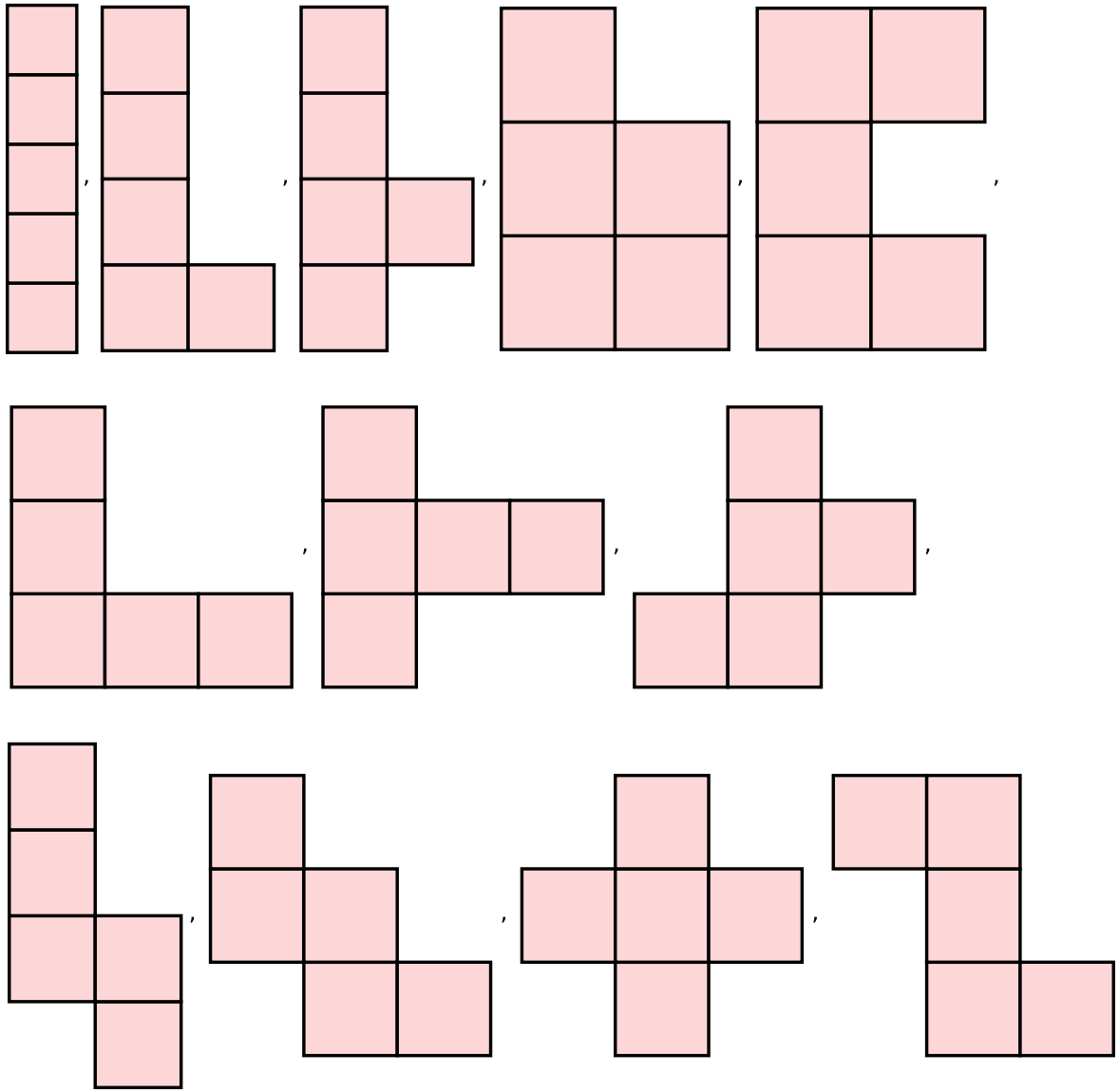
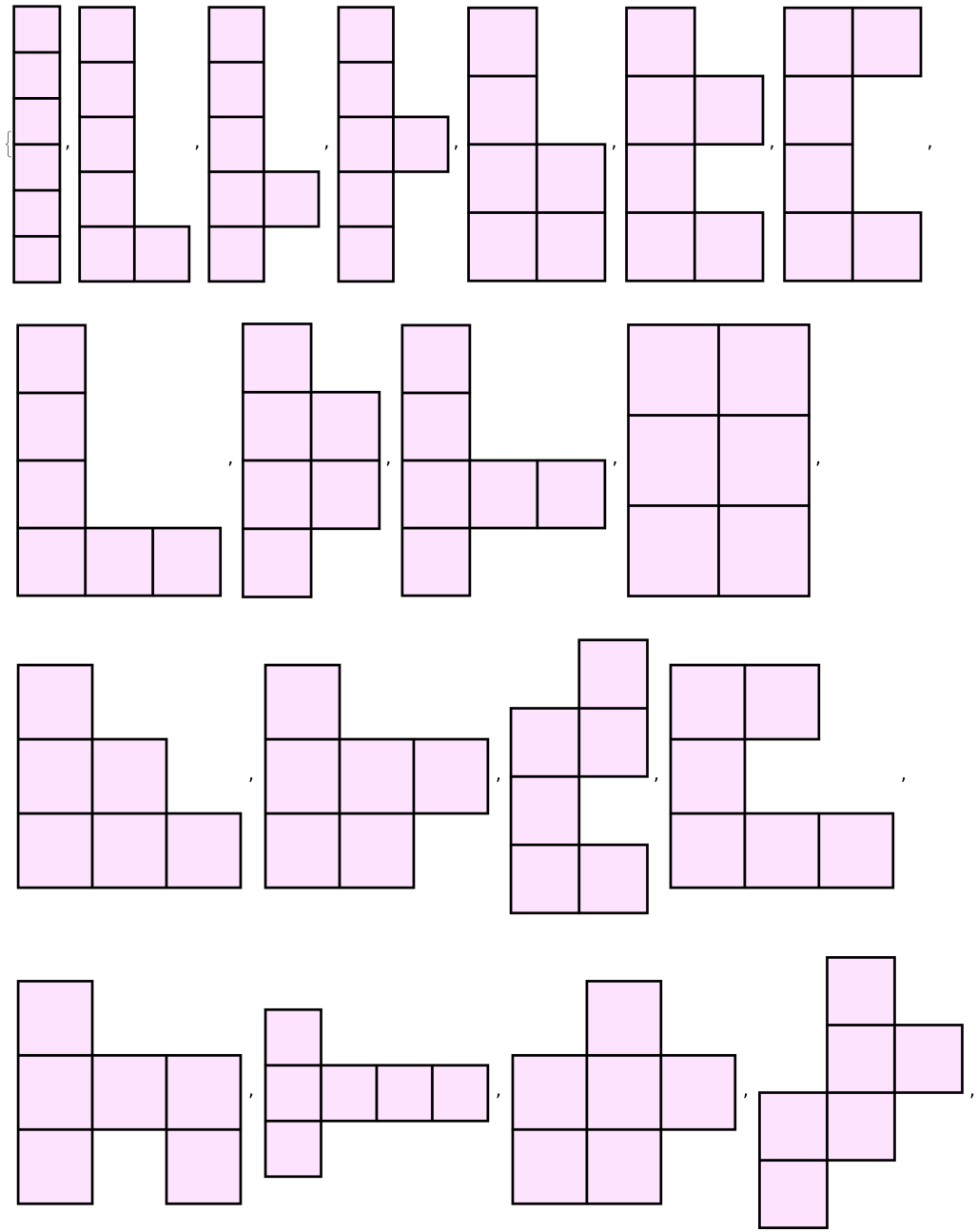
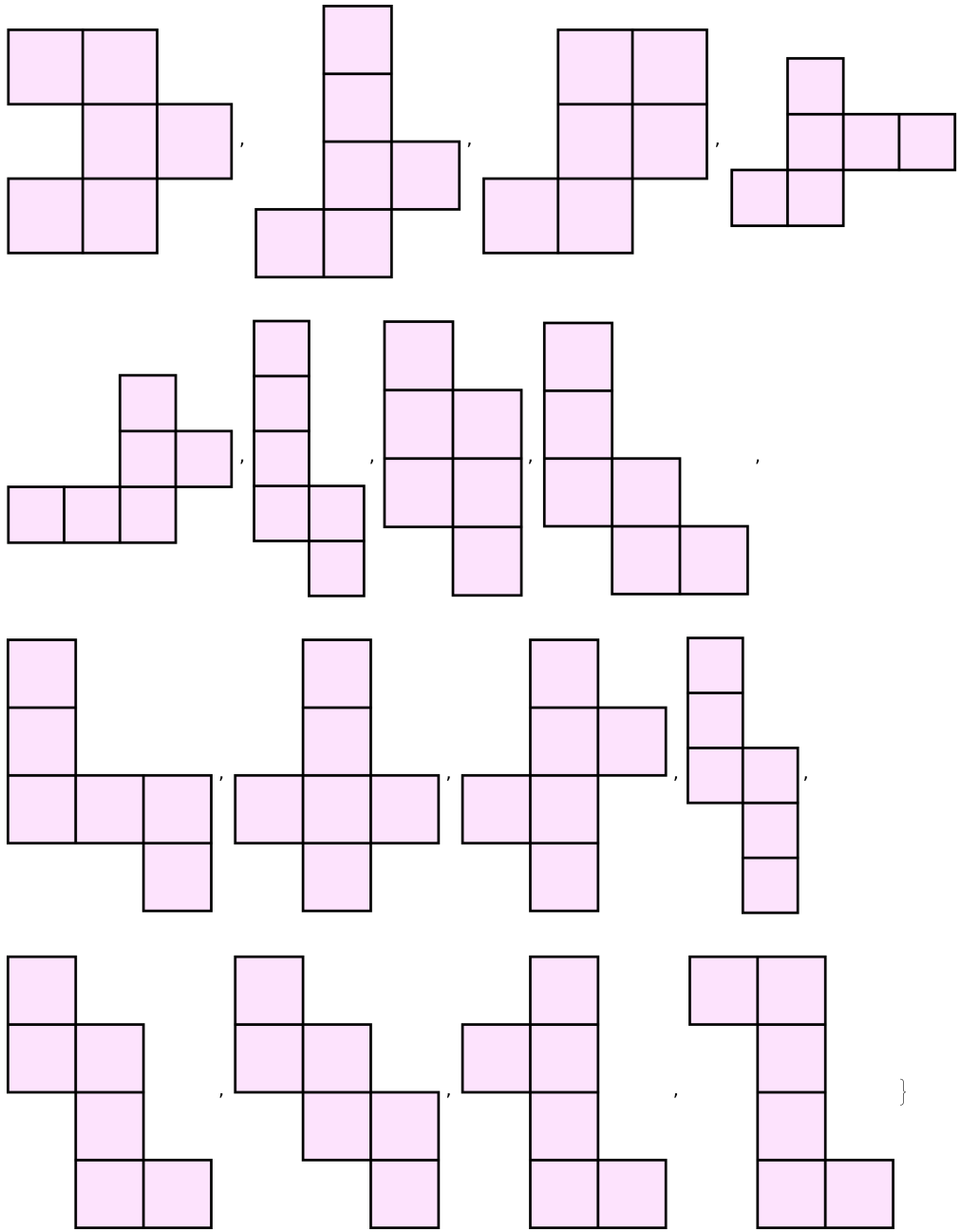


Free Hexominoes I

Free Hexominoes I



Free Hexominoes II



F-tiling

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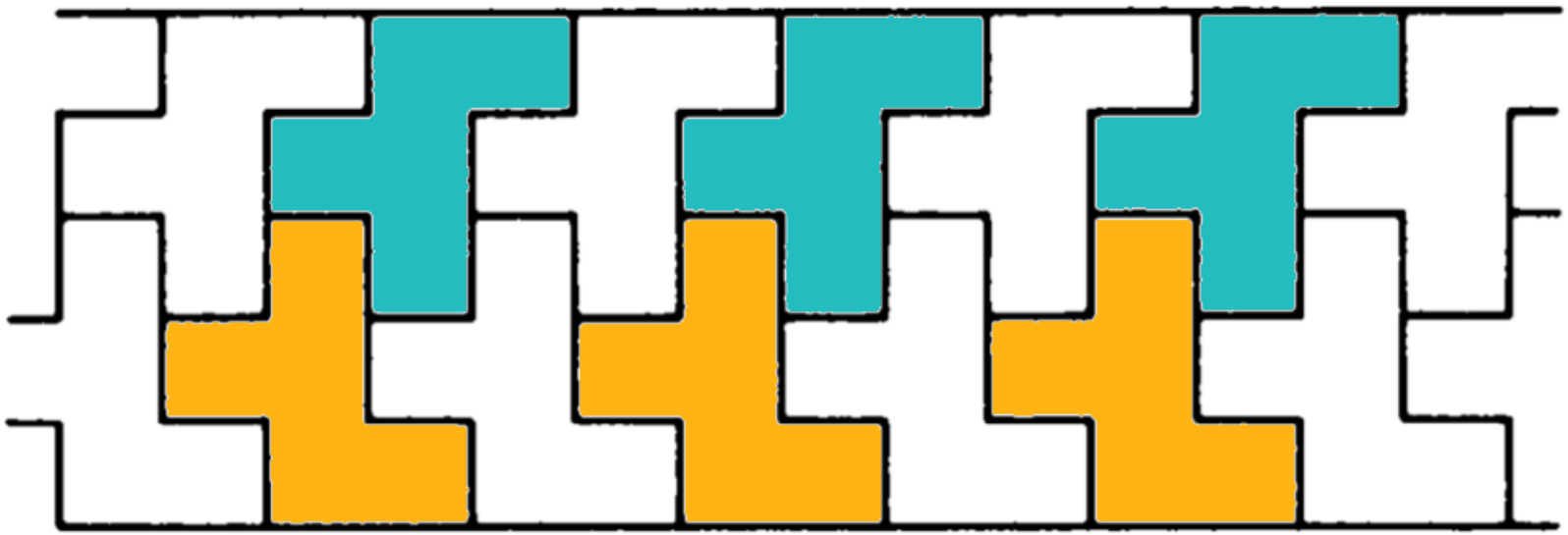
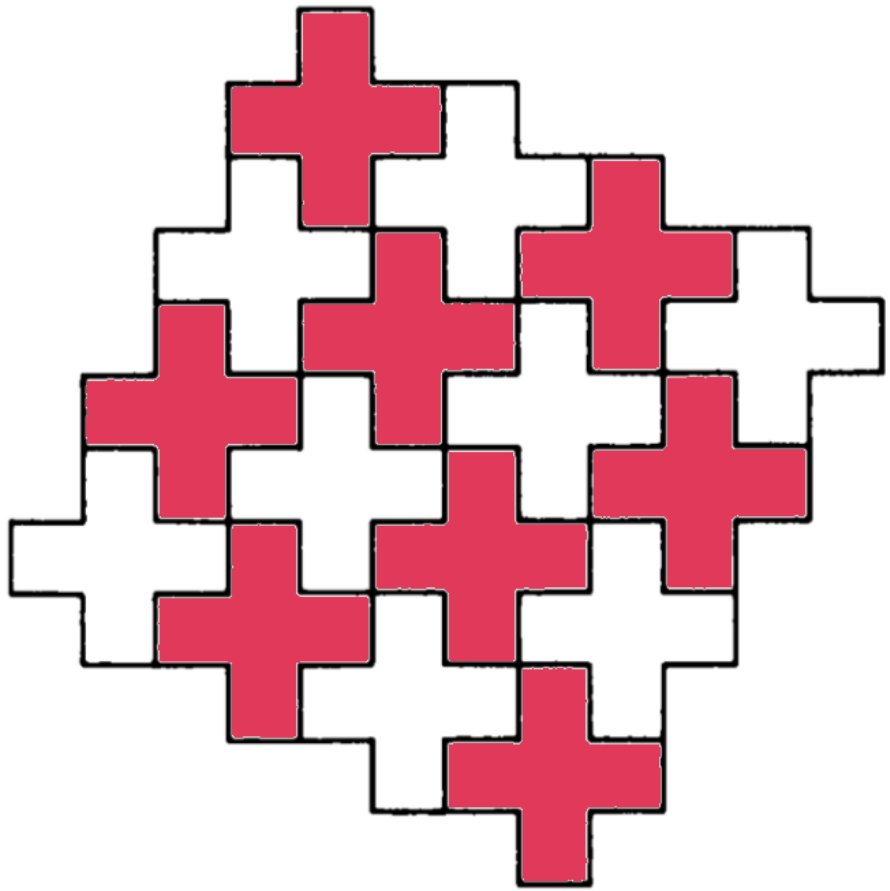


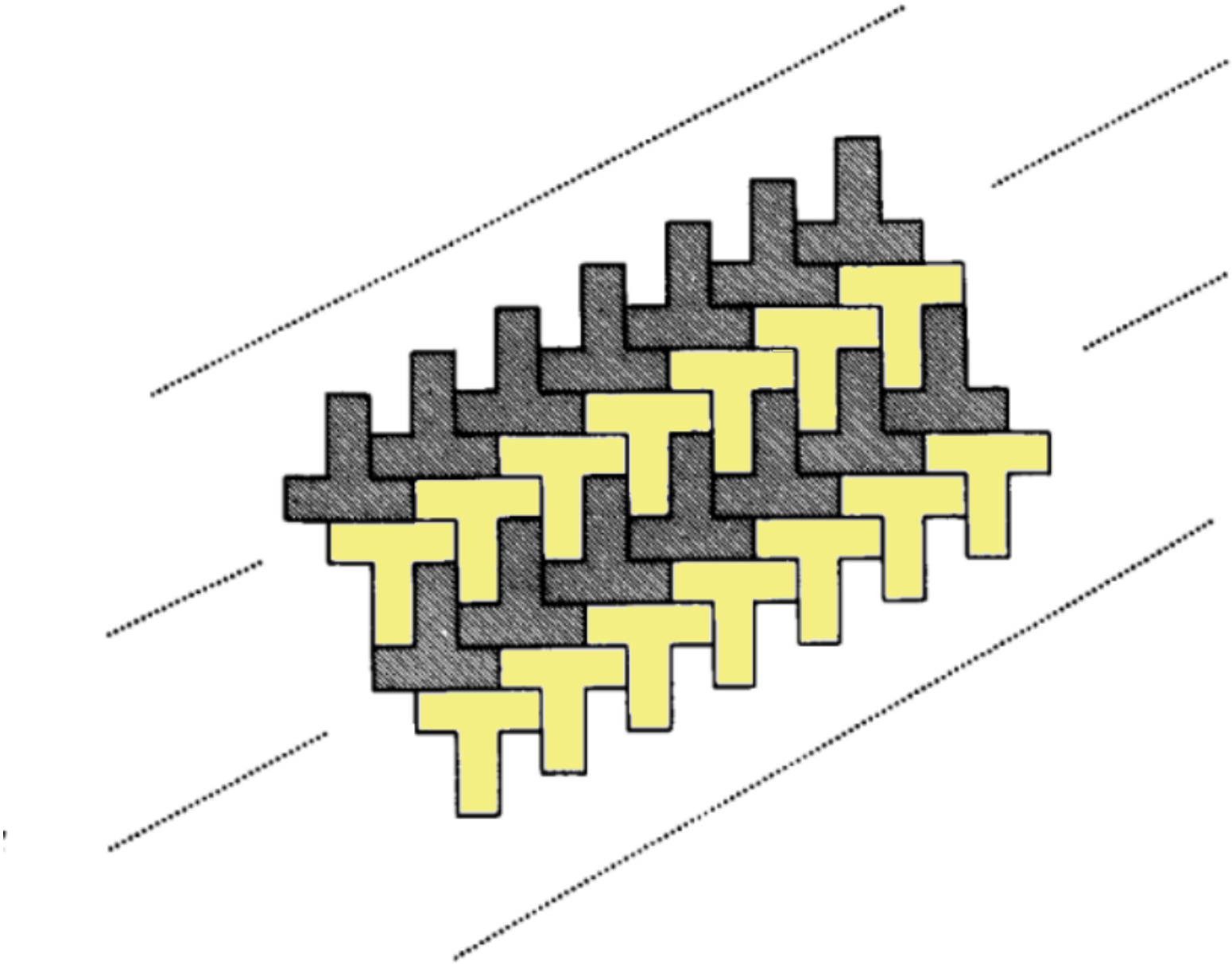
X-tiling

X-tiling



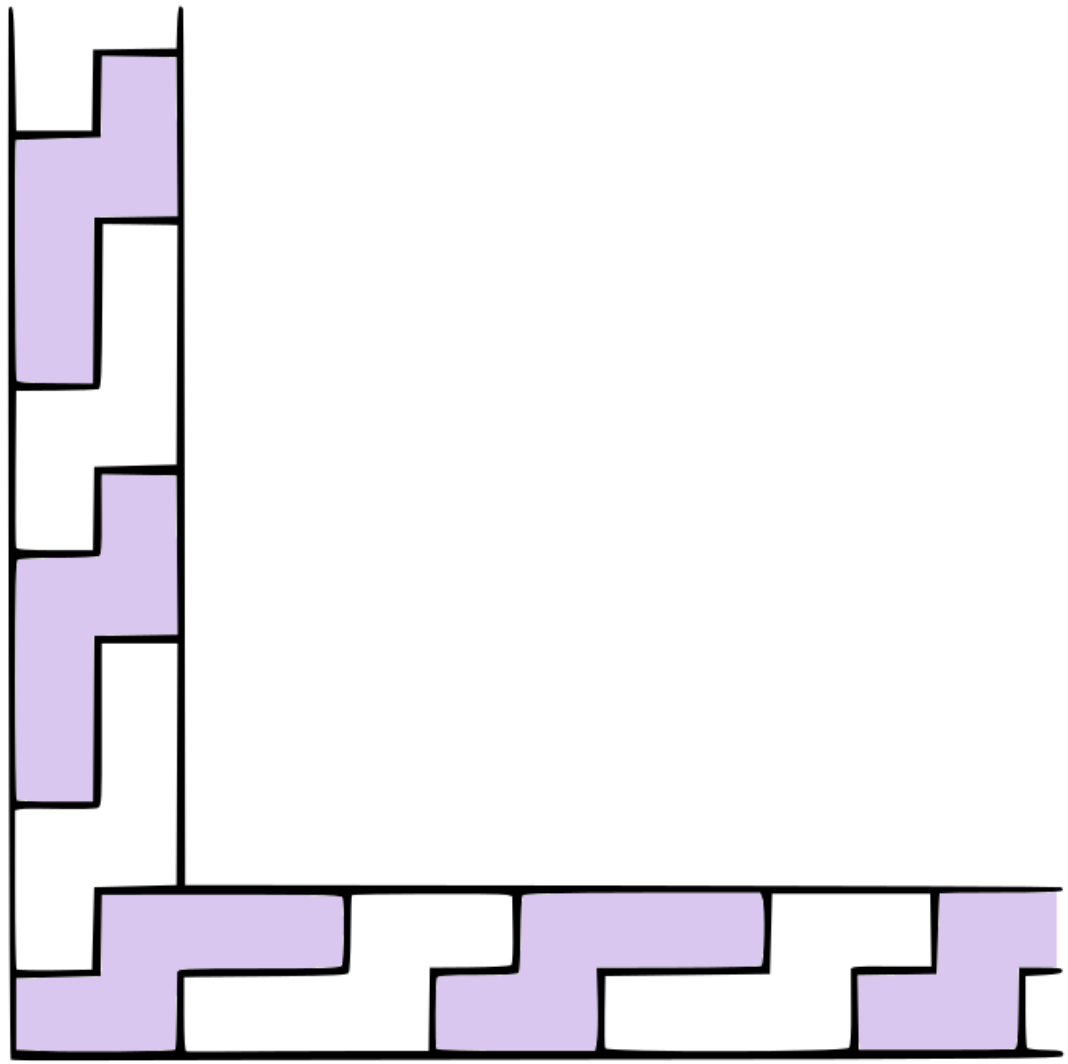
T-tiling

T-tiling



N-tiling

N-tiling



Pentominoes in the Square

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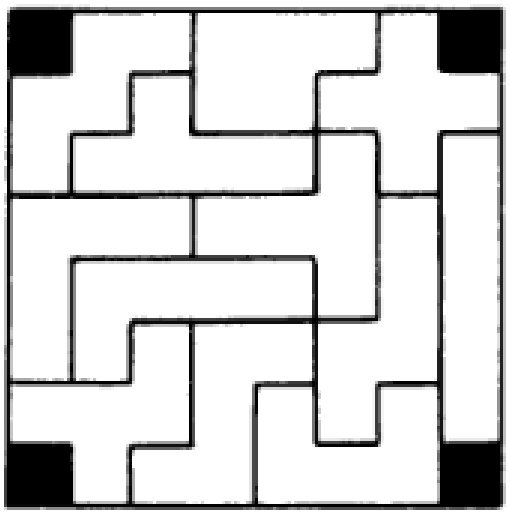
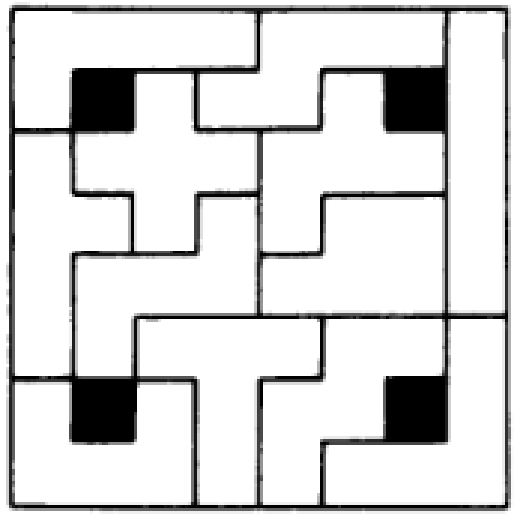
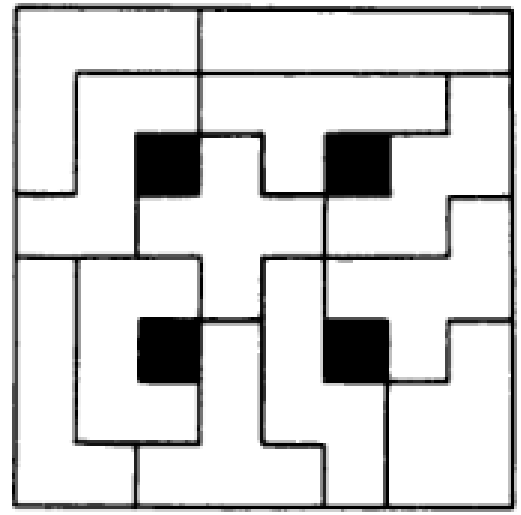
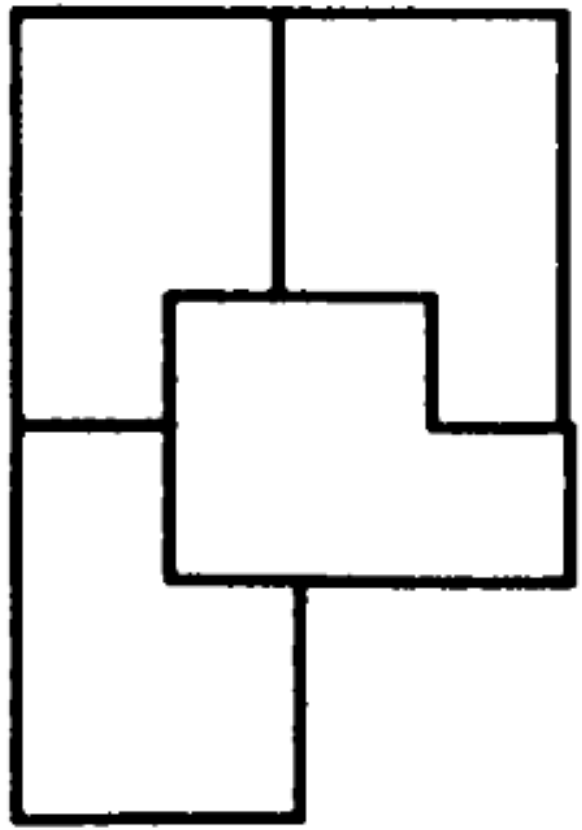




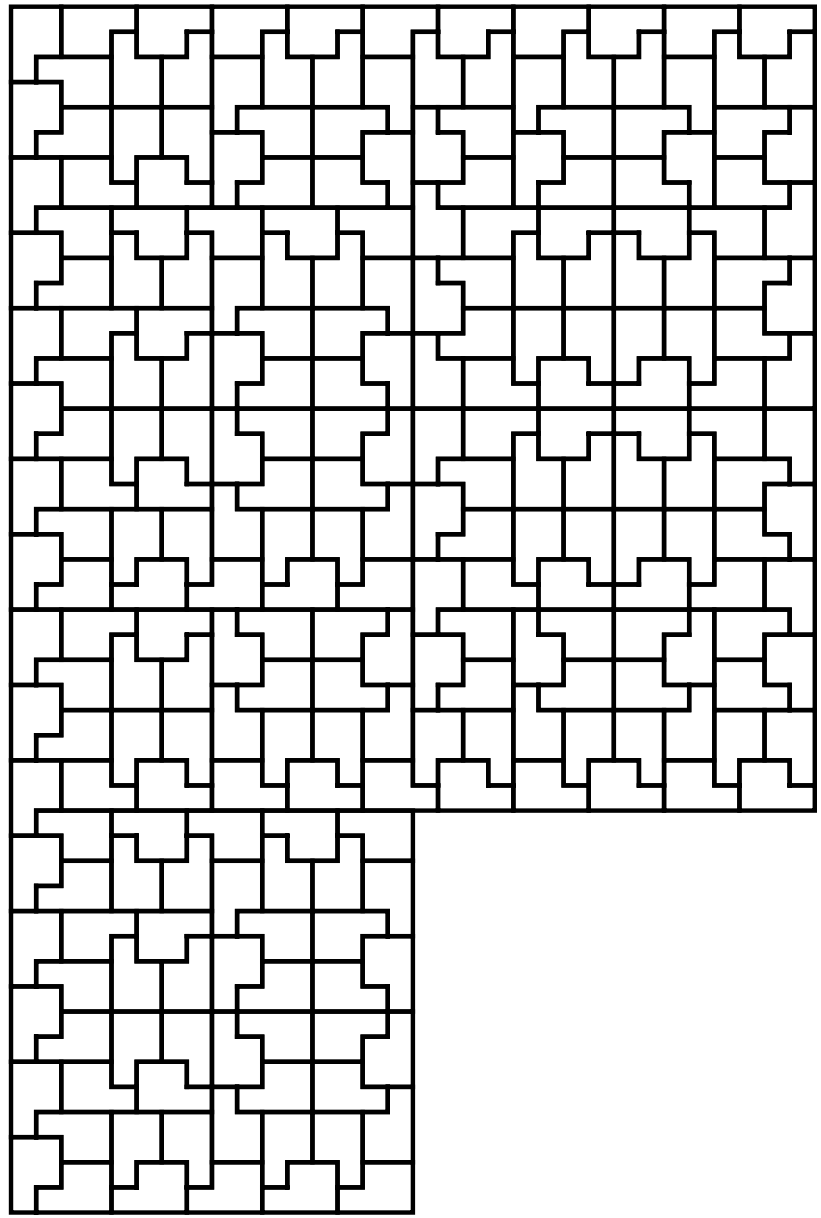


Rep-tiles

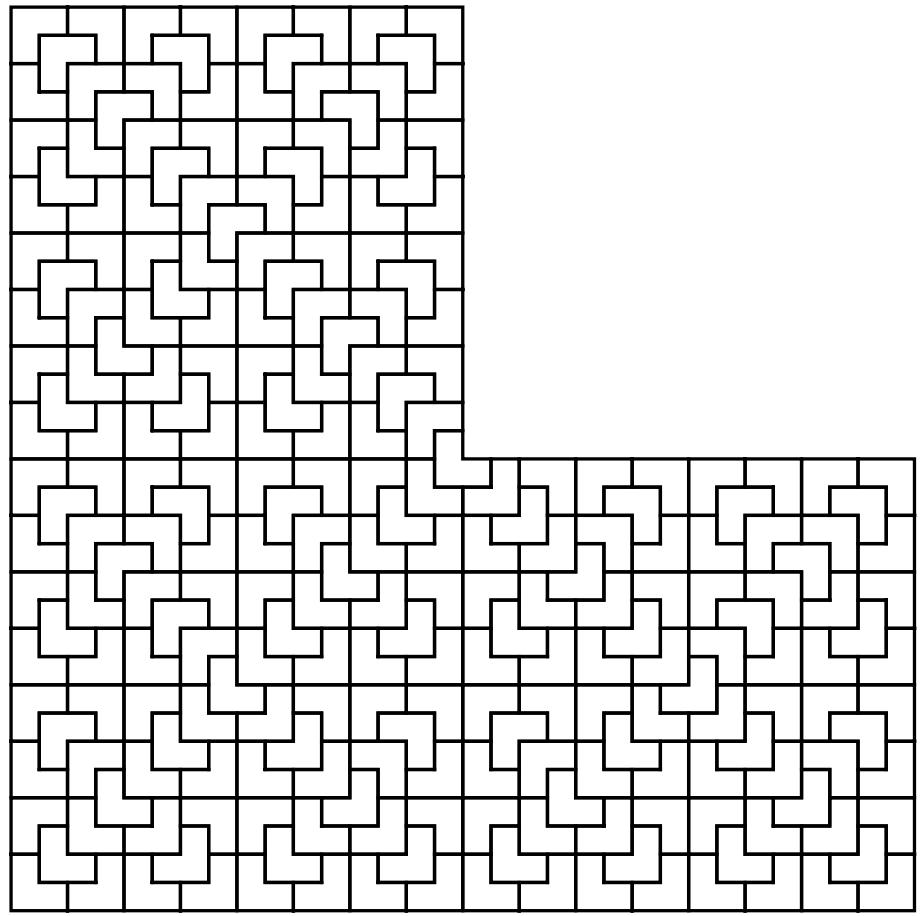
Rep-tiles



Rep-tiles: P



Rep-tiles: L-tromino



Huge Examples

Huge Examples

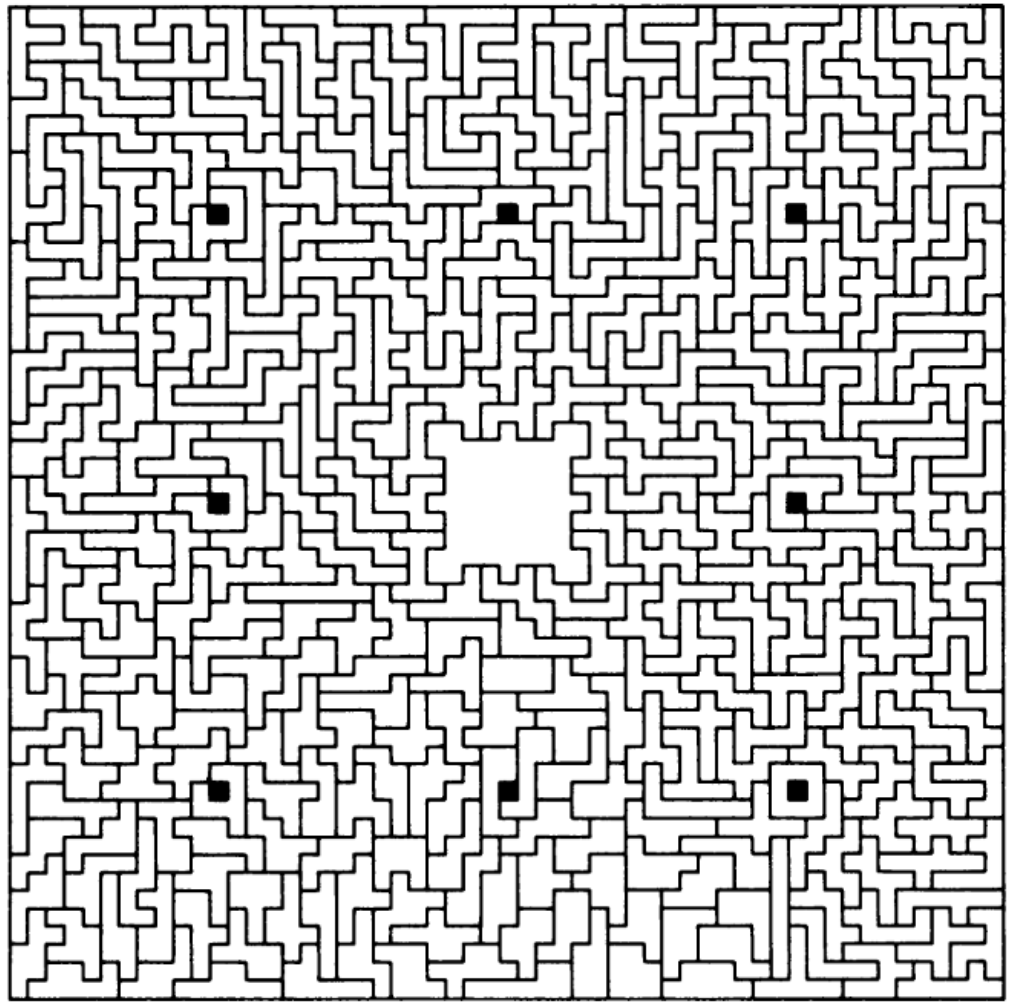


Figure: The 369 octominoes in a pattern of maximum symmetry. Solid internal corners. No crossroads.

Huge Examples II

Huge Examples II

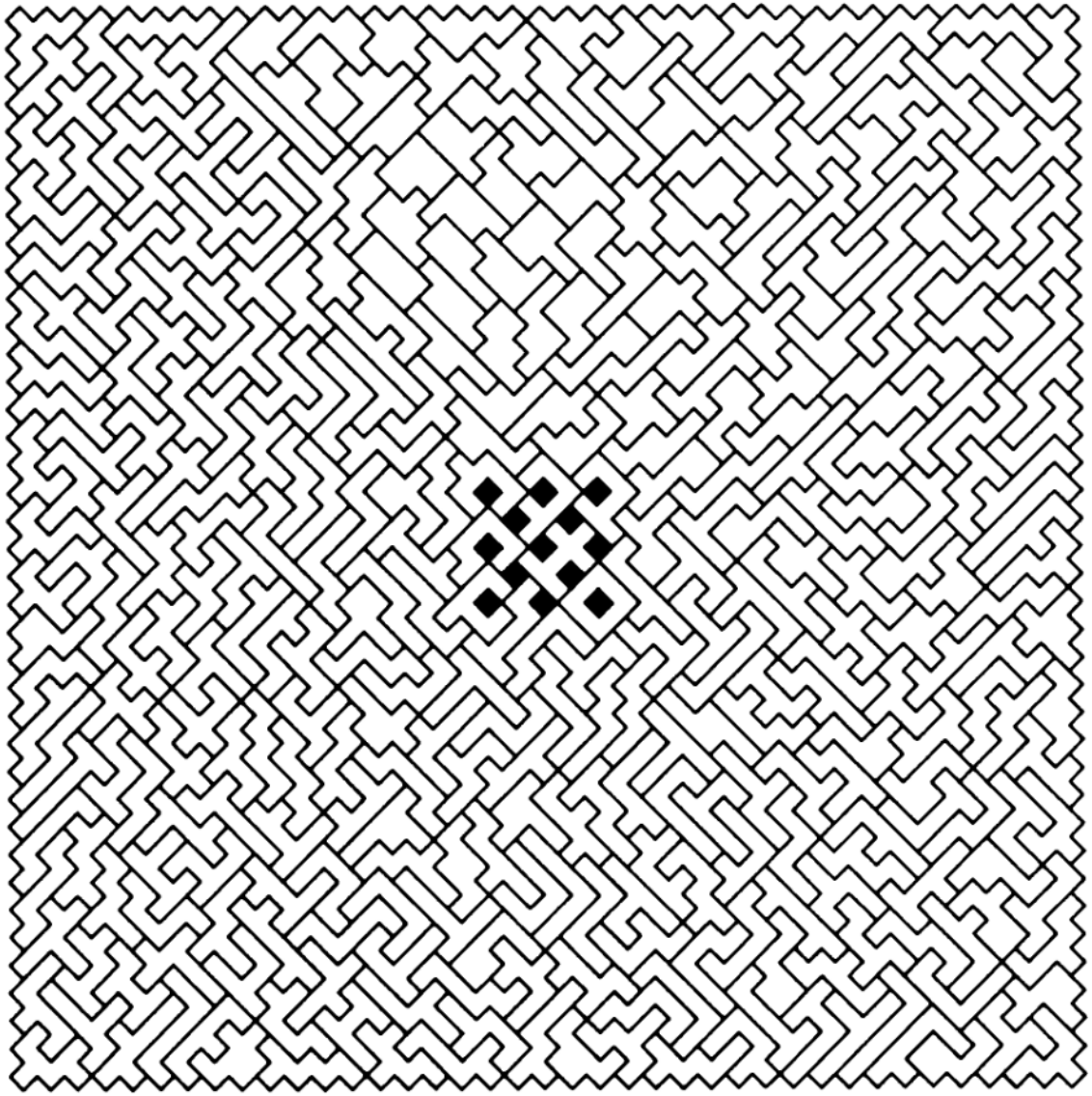


Figure: The 369 octominoes.

Huge Examples III

Huge Examples III

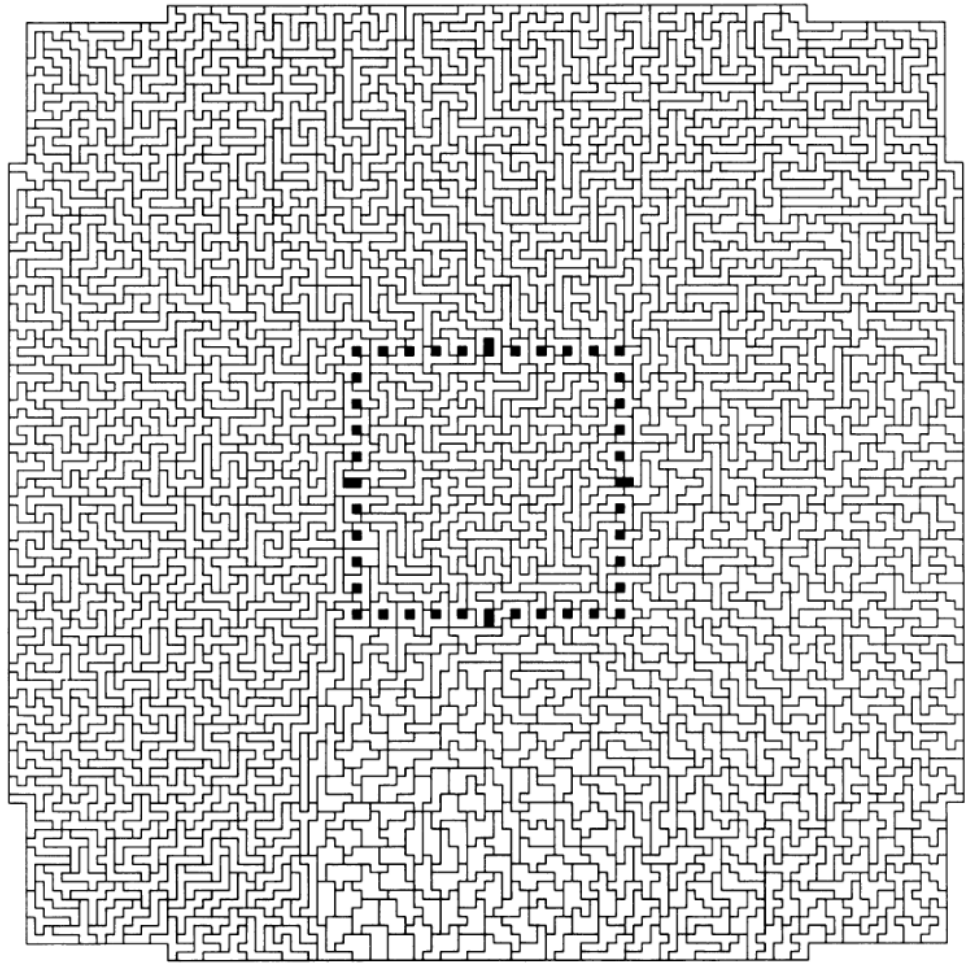


Figure: The 1285 enneominoes in a truncated 109 x 109 square.

Blokus

Thanks!

Thanks!

- Wikimedia Commons
- OEIS
- Golomb, Solomon W. *Polyominoes: puzzles, patterns, problems, and packings.* Princeton University Press, 1996.
- Rangel-Mondragón, J. *Polyominoes and related families.* Mathematica Journal, 9(3), pp.609-640, 2004.