Pocketable Mathematical Problems

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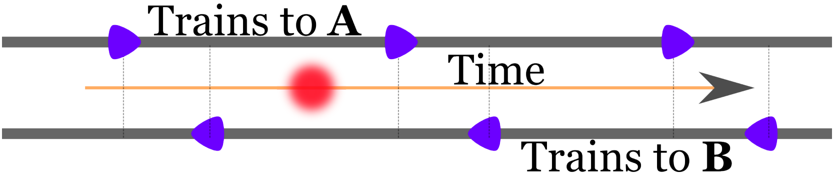
A Commuter on the Platform

A commuter noticed that whenever she arrives to the subway station next to her house, the first train would be leaving for **A** *five* times more often than for **B**. She knows, however, that trains in both directions are running with equal intervals.

Can you explain this phenomenon?

Solution

The platform viewed in time:

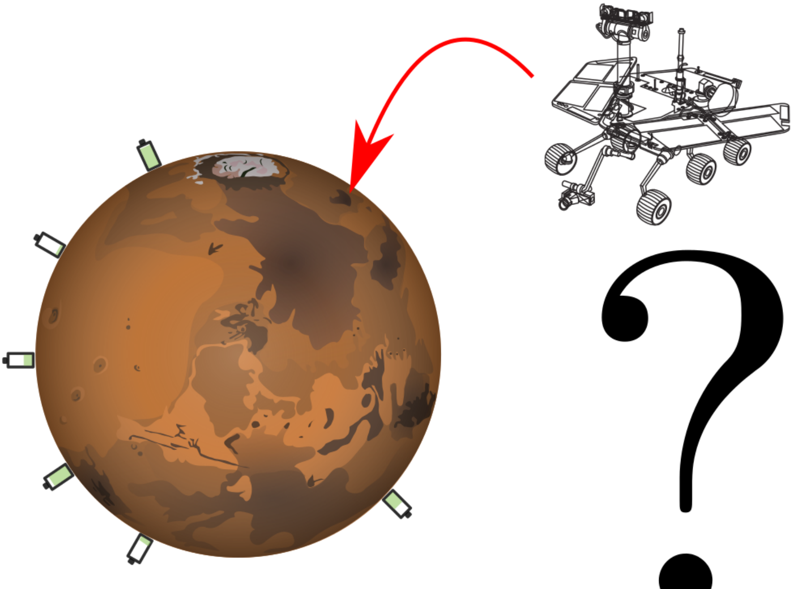


Driving Around Mars

The previous Mars mission has left a collection of batteries for a Mars rover along the equator. The total charge of these batteries is sufficient to ride around the planet. *Show* that the current mission can pick one of the equatorial batteries and drop a fully discharged rover on it, so that the rover will then be able to circle the equator using only the charge of the distributed batteries.

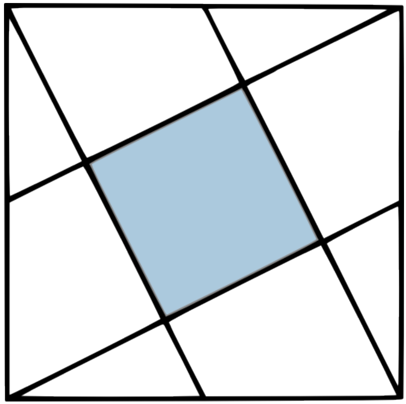
Driving Around Mars (not to scale)

Images: Wikimedia Commons

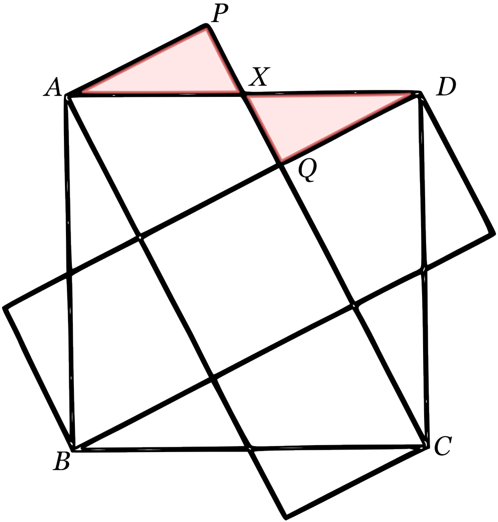


Square Within a Square

One draws lines from the vertices of a square to the midpoints of the sides, as shown. *Prove* that the area of the smaller square is **1/5** of the given one.



Solution

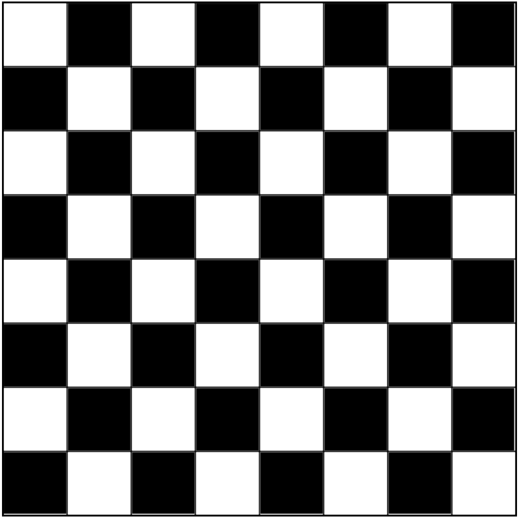


$APX \cong XQD$

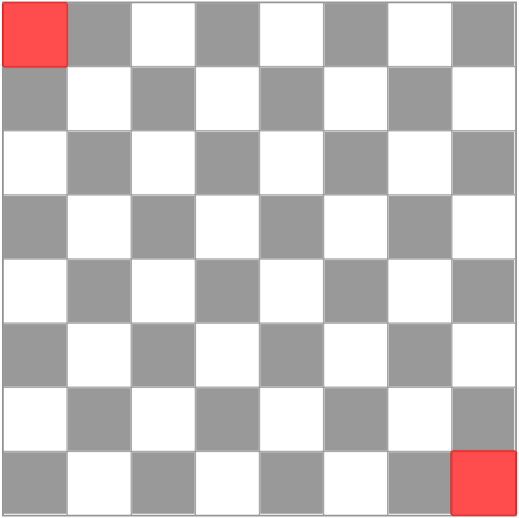
Dominoes on a Chessboard

One removes two opposite corner squares from a usual chessboard. Can the remaining part of it be covered by 2×1 dominoes without overlaps?

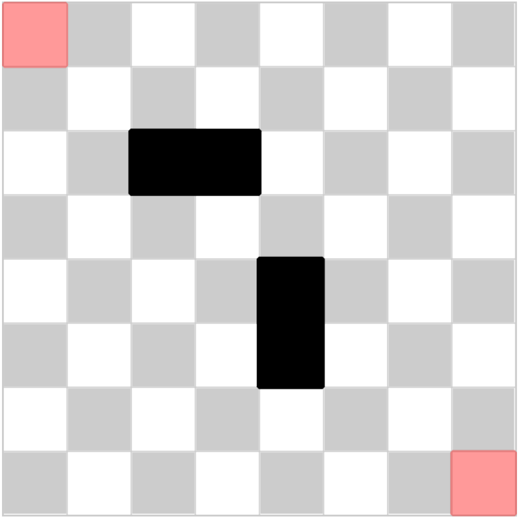
Dominoes on a Chessboard



Dominoes on a Chessboard



Dominoes on a Chessboard



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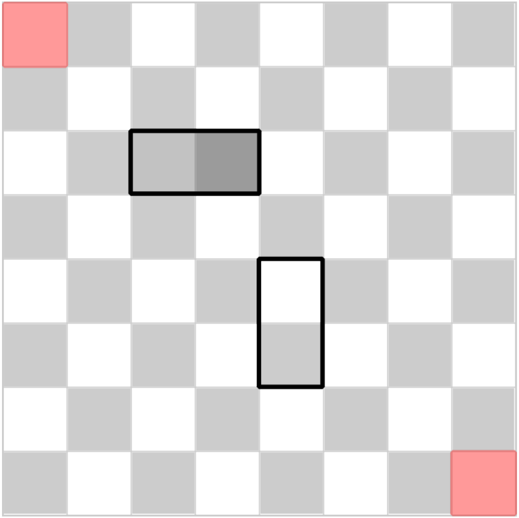

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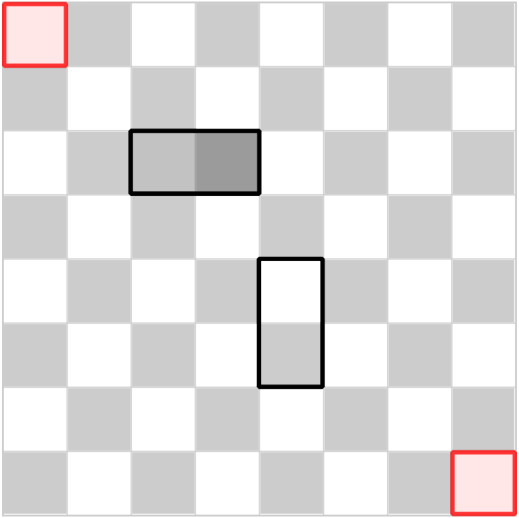

The Ordered Partitions of n

The number 3 can be expressed as the sum of one or more natural numbers in 4 ways, taking into account the order of the terms:

$$3, \quad 1+2, \quad 2+1, \quad 1+1+1.$$

How many such expressions are there for the number n?

Solution

Consider n as a string of 1's

Solution

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$$n = \underbrace{1\,1\,1\,1\,1\,1\,1\,1\,\ldots\,1\,1}_{n \text{ times}}$$

Solution

$$n = 1\,1 \Big|\; 1\,1\,1 \Big| 1 \Big|\;\; 1\,1\,\ldots\,1\,1$$

Solution

$$
\begin{array}{l}
n = 1\,1 \,\Big|\; 1\,1\,1 \,\Big|\, 1 \,\Big|\; 1\,1\,\ldots\,1\,1 \\
n = 2 + \quad 3 + 1 + \quad n - 6
\end{array}
$$

Solution

$$n = 1\ 1 \mid \ 1\ 1\ 1 \mid 1 \mid \quad 1\ 1\ \ldots\ 1\ 1$$
$$n = 2 + \quad 3 + 1 + \quad n - 6$$

$n - 1$ spaces between 1's

Solution

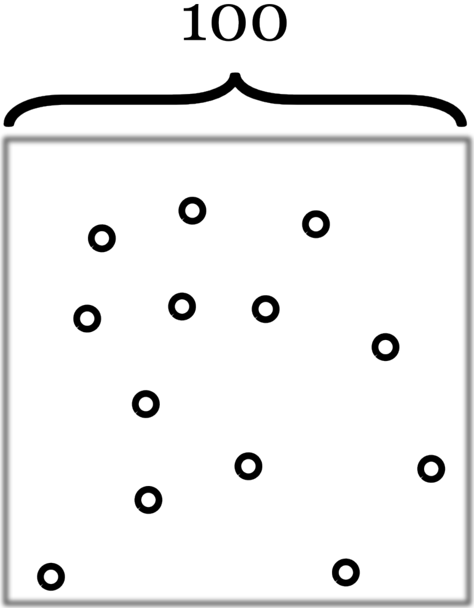
$$n = 1\,1 \mid 1\,1\,1 \mid 1 \mid 1\,1 \ldots 1\,1$$
$$n = 2 + 3 + 1 + n - 6$$

$n-1$ spaces between 1's, so $\mathbf{2^{n-1}}$ ways to place dividers.

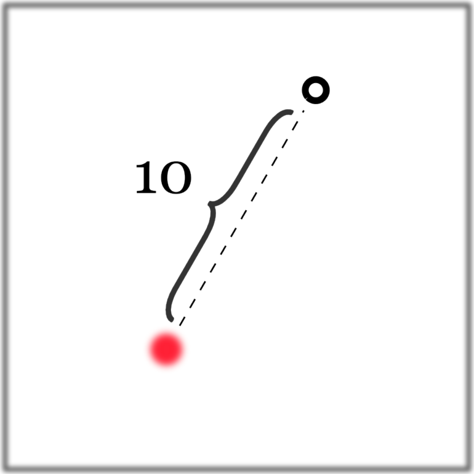
Counting Trees in a Forest

A square forest of size 100×100 with cylindrical trees of radius 1 is such that one sees at most at the distance 10 in any direction through it. *Prove* that there are at least 400 trees in this forest.

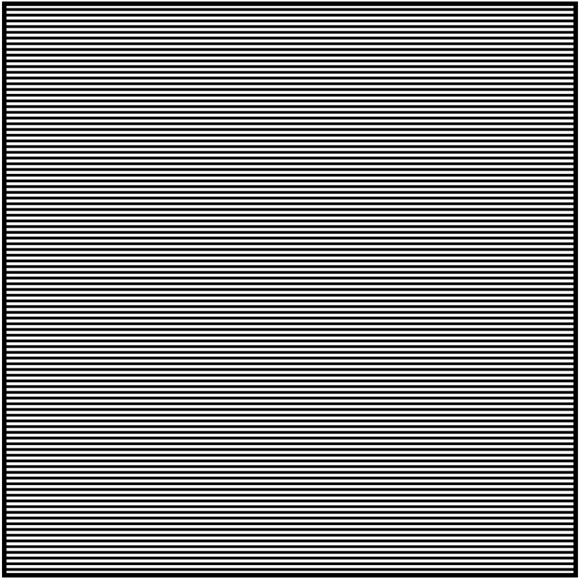
Counting Trees in a Forest



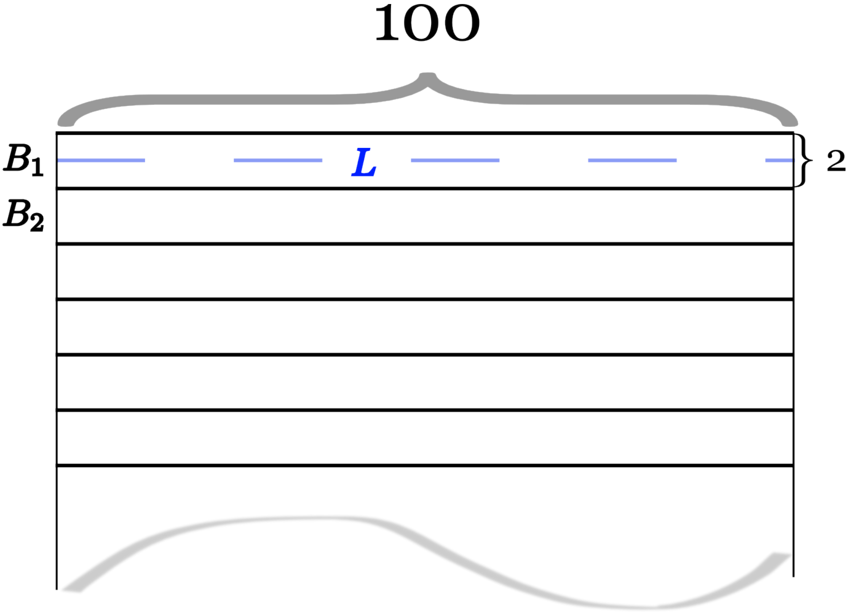
Counting Trees in a Forest



Solution



Solution



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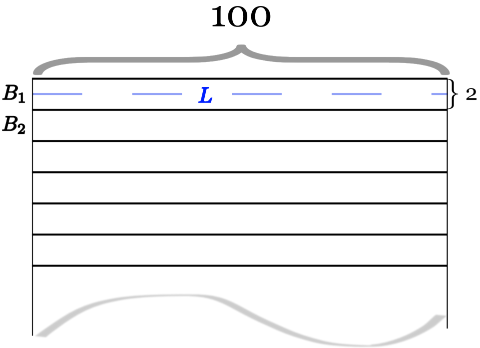
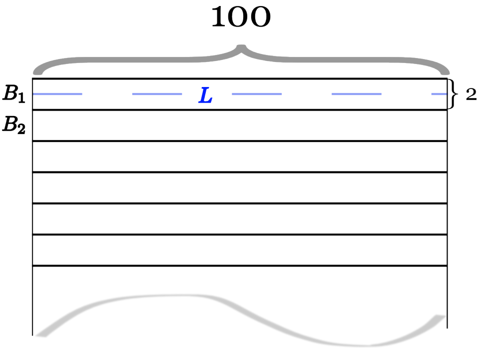


L is divided into at least 9 parts with trees:

$$8 \times 10 + 7 \times 2 < 100$$

Solution



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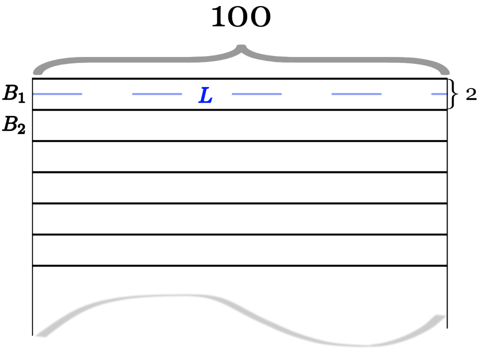
So band B_1 contains at least **8** trees.

Solution



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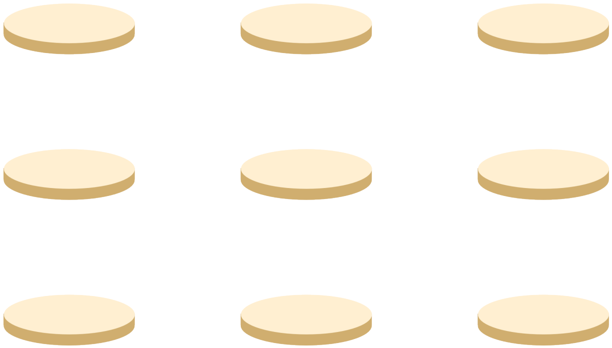
8 trees per band $\times$ 50 bands $=$ 400

Weighing Coins

How can you identify a single counterfeit coin, slightly lighter than the rest, from a group of **nine**, in only **two** weighings?

Solution



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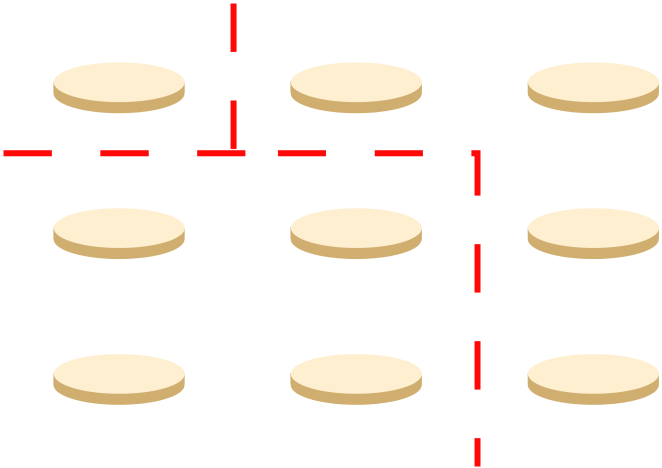

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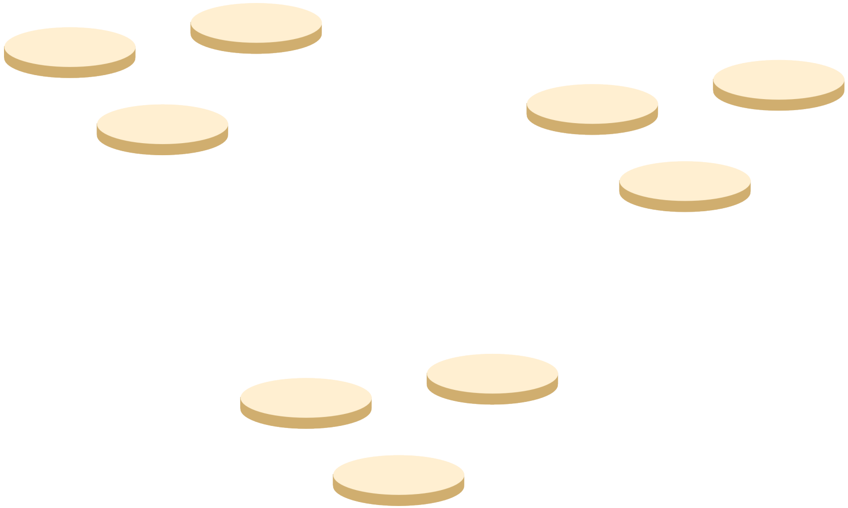

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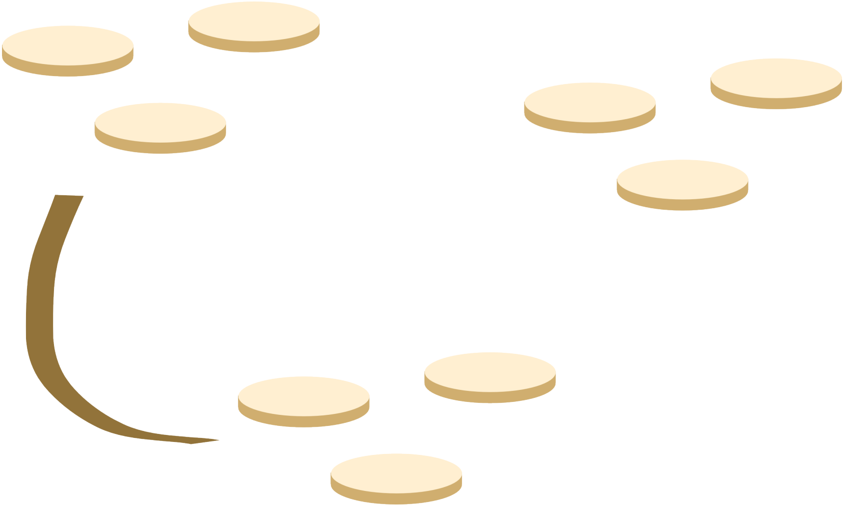

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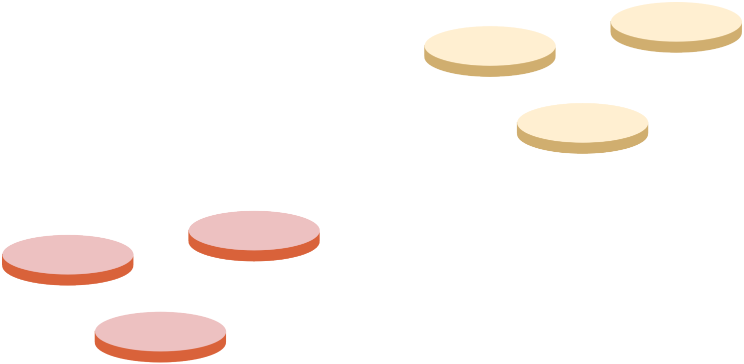

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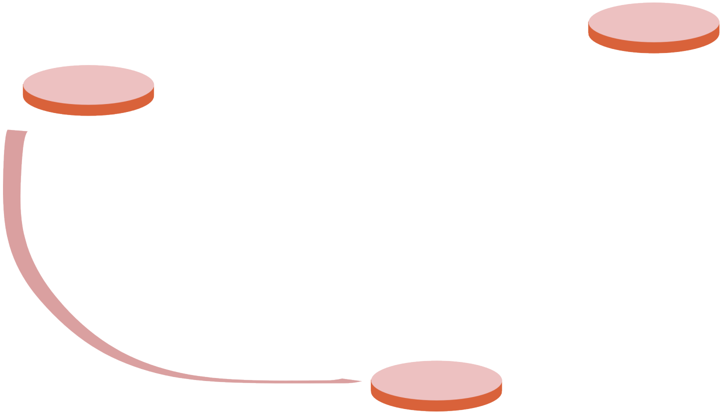

Thanks!

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An infinite checkered sheet of paper has numbers written in its squares so that every number is the mean of its four neighbours. We cut a rectangular table from the sheet. *Show* that the largest number in the table is adjacent to one of its sides.