USEFUL IDENTITIES WITH POWERS AND LOGARITHMS

1. Powers (here a and b are positive numbers; when r and s are integers, this assumption is not necessary)**.**

$$a^r \cdot a^s = a^{r+s} \qquad (ab)^r = a^r \cdot b^r$$

$$\frac{a^r}{a^s} = a^{r-s} \qquad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$(a^r)^s = a^{r \cdot s} \qquad ((ab)^r)^s = a^{r \cdot s} \cdot b^{r \cdot s}.$$

Since $\sqrt[n]{a} = a^{1/n}$, one has in particular:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

2. Logarithms (here a, b, u, v are positive numbers; $a \neq 1$ and $b \neq 1$)**.**
Recall the definition:

$$\log_b u = r \quad \text{if and only if} \quad b^r = u.$$

The properties of powers above give rise to the following properties of the logarithm:

$$\log_b(u \cdot v) = \log_b u + \log_b v$$

$$\log_b \left(\frac{u}{v}\right) = \log_b u - \log_b v$$

$$\log_b(u^r) = r \cdot \log_b u.$$

Converting between different bases:

$$\log_a u = \log_a b \cdot \log_b u.$$

Hint: this follows from $(a^r)^s = a^{r \cdot s}$. In particular, using the natural logarithm $\ln = \log_e$ with base $a = e$ gives

$$\ln u = \ln b \cdot \log_b u,$$

and so

$$\log_b u = \frac{\ln u}{\ln b}.$$